Higher Level Mathematics

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PART I

INTRODUCTORY TOPICS.

CHAPTER 1

QUADRATICS

1.1 FACTORISATION

QUESTIONS 1.1.

Q1. (1) $-6, -4$
(2) $-5, -3$
(3) $-6, 7$
(4) 3.5
(5) $4, -8$
(6) $16, -2$
(7) $4.41, 1.59$
(8) $5, 0$
(9) $0, 0.6$
(10) $-\frac{8}{3}, \frac{8}{3}$
(11) $\pm\frac{\sqrt{5}}{2}$
(12) $\pm\frac{\sqrt{10}}{2}$
(13) $1.5, -4.5$
(14) $-2, 8$
(15) $-1.5, 3.5$
(16) $2.4, -2$
(17) no solutions
(18) $\pm 1, \pm 2$
(19) -1.5
(20) $-2, -3$
(21) $7, 2$
(22) $-2, 5$
(23) $1, -4$
(24) $3, -3$
(25) -2
(26) 1
(27) $-2, -5$
(28) $5, 3$
(29) $-3, 7$
(30) $3, -6$
(31) $2, -2$
(32) $3, 3$
(33) -1

Q2. (1) $-\frac{3}{2}, -1$
(2) $-\frac{3}{2}, 2$
(3) $0.5, -3$
(4) $0.5, -1.5$
(5) $-1, 2.5$
(6) $-0.5, -3$
(7) $3, \frac{1}{3}$
(8) $-0.4, -3$
(9) $2, 1.5$
(10) $0.5, -3$
(11) $-\frac{2}{3}, 1$
(12) $4.5, 0.5$
(13) $-2, \frac{1}{3}$
(14) $0.5, -1$
(15) no solutions
(16) $-0.6, -1$
(17) $3, \frac{2}{3}$
(18) $-1, \frac{3}{2}$
(19) $-\frac{1}{3}, 2$
(20) $\frac{4}{3}, \frac{1}{3}$
(21) $1, -0.5$
(22) $\frac{2}{3}, -1$
(23) no solutions
(24) $\frac{8}{3}, \frac{2}{3}$
(25) $0.4, -3$
(26) $0, \pm 3$
(27) 2
(28) $-\frac{5}{2}, 1, \frac{5}{2}$
(29) $-4, -1, 3$

1.2 COMPLETING THE SQUARE

QUESTIONS 1.2.

Q3. (1) $0.24, -4.24$
(2) $-1.59, -4.41$
(3) $6.79, 2.21$
(4) $4.68, 0.32$
(5) $1.77, 0.566$
(6) no solutions
(7) $-0.59, -1.7$
(8) no solutions

1.3 QUADRATIC FUNCTION

QUESTIONS 1.3.

Q4. (1) $0.24, -4.24$
(2) $-1.59, -4.41$
(3) $6.79, 2.21$
(4) $4.68, 0.32$
(5) $1.77, 0.566$
(6) no solutions
(7) $-0.59, -1.7$
(8) no solutions

Q5. (1) ± 2
(2) $\pm\sqrt{6}$
(3) $\pm\frac{\sqrt{10}}{2}$
(4) $\pm 2, \pm 3$
(5) $\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{3}}$
(6) $\pm\sqrt{2}$
(7) $-1, 2$
(8) $-2, 2$

Q6. $(-1,-1),(4,4)$

Q7. $(-5,4),(-1,-4)$

Q8. $(1,1),(4,-2)$

Q9. $(-4,1),(1,-1.5)$

Q10. $(-4,12),(2,0)$

Q11. $(-2,5),(2,-3)$

Q12. $(-2,1),(2,-3)$

Q13. $y=-\frac{1}{2}x^2-x+4$

Q14. $y=\frac{7}{4}x^2+7x+4$

Q15. $0,4$

Q16. $-\frac{2}{7},2$

Q17. $-2\sqrt{2}<m<2\sqrt{2}$

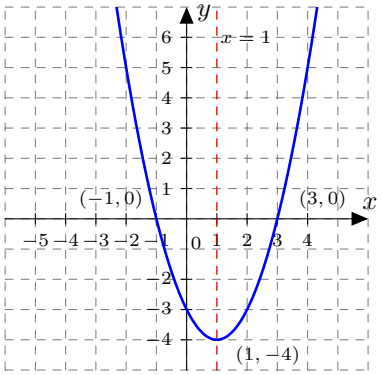
Q18. $c=3$, zeroes: $\frac{3}{2}\pm\frac{\sqrt{3}}{2}$

Q19.

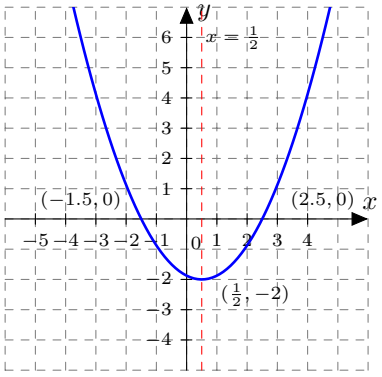
	$y=$	$y=$	$y=$			
(1)	$x^2-2x-15$	$(x-1)^2-16$	$(x-5)(x+3)$	$-3,5$	$(1,-16)$	64
(2)	$2x^2-12x+10$	$2(x-3)^2-8$	$2(x-1)(x-5)$	$1,5$	$(3,-8)$	64
(3)	$2x^2+5x-3$	$2(x+\frac{5}{4})^2-\frac{49}{8}$	$2(x-\frac{1}{2})(x+3)$	$-3,\frac{1}{2}$	$(-\frac{5}{4},-\frac{49}{8})$	49
(4)	x^2+4x+1	$(x+2)^2-3$	$(x+2-\sqrt{3})(x+2+\sqrt{3})$	$-2\pm\sqrt{3}$	$(-2,-3)$	12
(5)	$-2x^2-18x-28$	$-2(x+\frac{9}{2})^2+\frac{25}{2}$	$-2(x+2)(x+7)$	$-2,-7$	$(-\frac{9}{2},\frac{25}{2})$	100
(6)	$-4x^2+28x-49$	$-4(x-\frac{7}{2})^2$	$-4(x-\frac{7}{2})^2$	3.5	$(\frac{7}{2},0)$	0
(7)	$-\frac{1}{2}x^2-2x-3$	$-\frac{1}{2}(x+2)^2-1$	—	none	$(-2,-1)$	-2
(8)	$-4x^2+8x-2$	$-4(x-1)^2+2$	$-4(x-1+\frac{\sqrt{2}}{2})(x-1-\frac{\sqrt{2}}{2})$	$1\pm\frac{\sqrt{2}}{2}$	$(1,2)$	32
(9)	$-2x^2+2x\sqrt{2}+7$	$-2(x-\frac{\sqrt{2}}{2})^2+8$	$-2(x-\frac{\sqrt{2}}{2}+2)(x-\frac{\sqrt{2}}{2}-2)$	$\frac{\sqrt{2}}{2}\pm2$	$(\frac{\sqrt{2}}{2},8)$	64

Q20. (1) 3 (2) 16 (3) 8 (4) 11 (5) 6 (6) 2 (7) 7 (8) 14 (9) 10 (10) 12

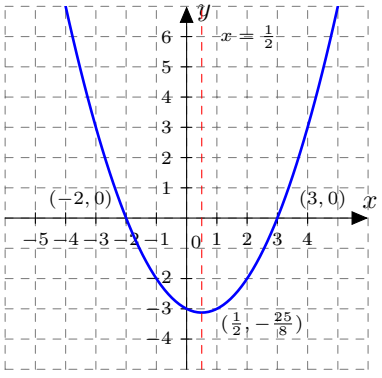
Q21. (1)



(2)



(3)



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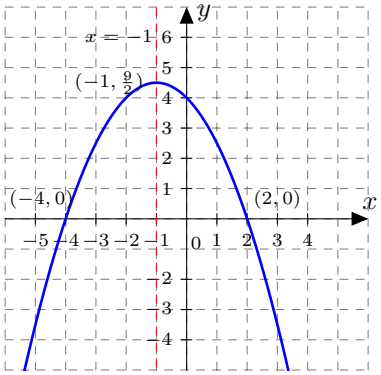

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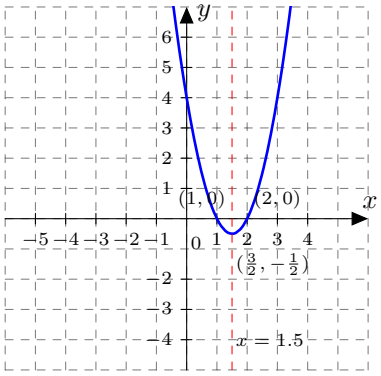
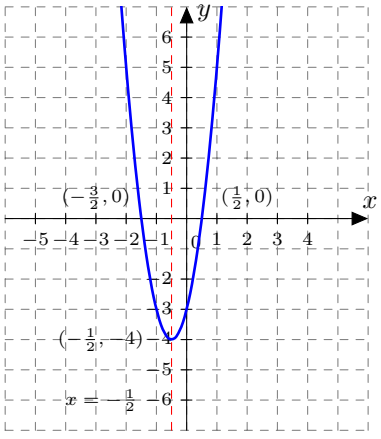


(6)



1.4 Quadratic inequalities

Questions 1.4.

Q22. (1) no solutions
(2) $x < -2$ or $x > 0$
(3) $x \in \mathbb{R}$
(4) $x < -1$ or $x > 3$
(5) $1 \leq x \leq 5$
(6) $6 < x < -4$
(7) $-4 \leq x \leq 6$
(8) $x < -3$ or $x > \frac{3}{2}$
(9) $x \leq -\frac{5}{2}$ or $x \geq \frac{2}{3}$
(10) $\frac{5}{3} \leq x \leq 3$
(11) $x \leq -4$ or $x \geq -2$
(12) $\frac{1}{3} < x < 5$

Q23. (1) $x < -2$ or $x > 1$
(2) $-4 \leq x \leq -1$
(3) $-\frac{2}{3} < x < \frac{3}{2}$
(4) $x < 1$ or $x > 2.5$
(5) $-3 \leq x \leq 0$
(6) $x < 0$ or $x > 5$
(7) $-3 \leq x \leq 2$
(8) $x \leq 2$ or $x \geq 3$
(9) $x \neq -3$
(10) $x \in \mathbb{R}$
(11) no solutions
(12) $x \neq 1$
(13) $x = \frac{1}{2}$
(14) $0 < x < \frac{2}{3}$
(15) $x < -2$ or $x > 1$
(16) $-2 \leq x \leq 0$
(17) $x < -2$ or $x > \frac{3}{2}$
(18) $x \leq \frac{1}{2}$ or $x \geq 3$
(19) no solutions
(20) $x < 0$ or $x > \frac{2}{5}$

1.5 Viette's formulae

Questions 1.5.

Q24.

	a	b	c	$\frac{c}{a}$	$-\frac{b}{a}$	x_1	x_2
1	+	−	+	+	+	+	+
2	+	?	−	−	?	+	−
3	+	?	−	−	?	−	+
4	+	+	+	+	−	−	−

	a	b	c	$\frac{c}{a}$	$-\frac{b}{a}$	x_1	x_2
5	−	+	−	+	+	+	+
6	−	?	+	−	?	+	−
7	−	?	+	−	?	−	+
8	−	−	−	+	−	−	−

Q25. (1) different signs
(2) both positive
(3) different signs
(4) different signs
(5) both negative
(6) different signs
(7) different signs
(8) both positive
(9) different signs
(10) different signs
(11) both negative
(12) both positive

Q26. $m < -6$

Q27. $m \in]0, 1[\cup]1, +\infty[$

Q28. $m < 3$

Q29. (1) different signs
(2) both positive
(3) different signs
(4) both positive
(5) both negative
(6) no solutions

Q30. (1) $\frac{5}{3}$
(2) -2
(3) $\frac{61}{9}$
(4) $\frac{395}{27}$
(5) $-\frac{5}{6}$
(6) $\frac{\sqrt{97}}{3}$
(7) $\frac{61}{36}$
(8) $-\frac{61}{18}$
(9) $\frac{5\sqrt{97}}{9}$

Q31. (1) $-\frac{8}{5}$
(2) $\frac{2}{5}$
(3) $\frac{44}{25}$
(4) $\frac{752}{125}$
(5) -4
(6) $\frac{2\sqrt{6}}{5}$
(7) 11
(8) $\frac{22}{5}$
(9) $\frac{16\sqrt{6}}{25}$

Q32. (1) $4x^2 + 7x - 8 = 0$
(2) $2x^2 - 7x - 16 = 0$
(3) $16x^2 - 113x + 64 = 0$
(4) $32x^2 - 113x + 32 = 0$
(5) $64x^2 - 1015x - 512 = 0$
(6) $8x^2 + 7x - 4$
(7) $64x^2 - 113x + 64 = 0$
(8) $32x^2 + 113x + 32 = 0$
(9) $2x^2 - 7x - 16 = 0$

Q33. (1) $-5x^2 - 3x + 9 = 0$
(2) $-5x^2 + 6x + 36 = 0$
(3) $25x^2 - 99x + 81 = 0$
(4) $100x^2 - 198x + 81 = 0$
(5) $125x^2 - 432x - 729 = 0$
(6) $9x^3 + 3x - 5 = 0$
(7) $81x^2 - 99x + 25 = 0$
(8) $45x^2 + 99x + 45 = 0$
(9) $45x^2 - 57x - 361 = 0$

1.6 Applications of quadratics

Questions 1.6.

Q34. *Hint:* when one side of a rectangle is x, the other is $20 - x$.

Q35. $10, 24, 26$

Q36. $6m \times 8m$

Q37. $15m \times 30m$

Q38. $17.6m \times 4.7m$

Q39. $12m \times 12m$

Q40. a) $16m$
b) after 6 seconds
c) after 2 seconds
d) $\frac{64}{3}m \approx 21.3m$

Q41. $5cm \times 7cm$

Q42. $5cm, 11cm$

Q43. $7cm \times 13cm$

Q44. $8cm, 15cm$

Q45. —

Q46. —

Q47. $\left(\frac{2}{3}, 0\right)$

Q48. $\left(-\frac{1}{3}, \frac{8}{9}\right)$

CHAPTER 2

LINES, CIRCLES AND VECTORS

QUESTIONS 2.1.

Q1. 54

Q2. (1) $y = -\frac{1}{2}x$
(2) $y = -\frac{2}{3}x - 1$
(3) $y = \frac{3}{2}x - 8$
(4) $5x + 2y + 1 = 0$
(5) $3x - 4y + 11 = 0$
(6) $2x + y - 3 = 0$

Q3. (1) $(4, 3), r = 6$
(2) $(-3, 1), r = \sqrt{11}$
(3) $(-8, 6), r = 10$
(4) $(-5, 0), r = 3\sqrt{2}$
(5) $(-2, \frac{1}{2}), r = \frac{5}{2}$
(6) $(\frac{3}{2}, -4), r = \frac{9}{2}$
(7) $(-\frac{7}{2}, \frac{5}{2}), r = 4$

Q4. (1) $(-4, -1), (3, 0)$
(2) $(-3, 4), (-10, -3)$

Q5. (1) $\sqrt{37} - \sqrt{13} \approx 2.48$
(2) $6\sqrt{2} - 2\sqrt{5} \approx 4.07$

Q6. 4

Q7. -7

Q8. $2q = 5p + 24$

Q9. $c = -6; (-3, 1)$ or $c = -26; (-5, 7)$

Q10. $\frac{5 \pm \sqrt{105}}{4}$

Q11. $-\frac{4}{3}$ or $\frac{3}{4}$

Q12. $\frac{1}{2}$ or 2

Q13. 12 or 22

QUESTIONS 2.2.

Q14. (1) $b - a$
(2) $b - c$
(3) $c - b$
(4) $c - a$
(5) $\frac{1}{2}b - a + \frac{1}{2}c$
(6) $-\frac{3}{4}a + \frac{1}{4}b + \frac{1}{2}c$
(7) $\frac{1}{2}a - \frac{1}{2}c$
(8) $\frac{3}{4}a + \frac{3}{4}b - \frac{1}{2}c$
(9) $\frac{1}{4}a + \frac{1}{4}b - \frac{1}{2}c$

Q15. (1) $\begin{pmatrix} anything \\ 5 \end{pmatrix}$
(2) $\begin{pmatrix} anything \\ 0 \end{pmatrix}$
(3) $\begin{pmatrix} -6 \\ anything \end{pmatrix}$

Q16. (1) $\begin{pmatrix} 0 \\ 6 \end{pmatrix}, \begin{pmatrix} 1 \\ 8 \end{pmatrix}, \begin{pmatrix} 2 \\ 10 \end{pmatrix}; \begin{pmatrix} t \\ 2t + 6 \end{pmatrix}, t \in \mathbb{R}$
(2) $\begin{pmatrix} 0 \\ 5 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \end{pmatrix}, \begin{pmatrix} 6 \\ 1 \end{pmatrix}; \begin{pmatrix} t \\ -\frac{2}{3}t + 5 \end{pmatrix}, t \in \mathbb{R}$
(3) $\begin{pmatrix} 0 \\ 3.5 \end{pmatrix}, \begin{pmatrix} 1 \\ 5 \end{pmatrix}, \begin{pmatrix} 2 \\ 6.5 \end{pmatrix}; \begin{pmatrix} 2t + 1 \\ 3t + 5 \end{pmatrix}, t \in \mathbb{R}$

Q17. (1) $\begin{pmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{pmatrix}$
(2) $\begin{pmatrix} -\frac{12}{13} \\ \frac{5}{13} \end{pmatrix}$
(3) $\begin{pmatrix} -\frac{1}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{5}}{5} \\ -\frac{2\sqrt{5}}{5} \end{pmatrix}$
(4) $\begin{pmatrix} \frac{3}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} \end{pmatrix}$
(5) $\begin{pmatrix} -1/\sqrt{5} \\ 2/\sqrt{5} \end{pmatrix}$
(6) $\begin{pmatrix} 0.626 \\ -0.780 \end{pmatrix}$

Q18. (1) $\begin{pmatrix} 1.8 \\ -2.4 \end{pmatrix}$
(2) $\begin{pmatrix} -24 \\ 10 \end{pmatrix}$
(3) $\begin{pmatrix} -\sqrt{3} \\ -2\sqrt{3} \end{pmatrix}$
(4) $\begin{pmatrix} 3\sqrt{6} \\ \sqrt{6} \end{pmatrix}$
(5) $\begin{pmatrix} -2\sqrt{3} \\ 4\sqrt{3} \end{pmatrix}$
(6) $\begin{pmatrix} 3.13 \\ -3.90 \end{pmatrix}$

Q19. (1) $\vec{v} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}, \vec{u} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$

(2) $\vec{v} = \begin{pmatrix} 4\sqrt{5} \\ -2\sqrt{5} \end{pmatrix}, \vec{u} = \begin{pmatrix} 2\sqrt{5} \\ 4\sqrt{5} \end{pmatrix}$

(3) $\vec{v} = \begin{pmatrix} \sqrt{10} \\ -3\sqrt{10} \end{pmatrix}, \vec{u} = \begin{pmatrix} 3\sqrt{10} \\ \sqrt{10} \end{pmatrix}$

(4) $\vec{v} = \begin{pmatrix} 2\sqrt{5} \\ -4\sqrt{5} \end{pmatrix}, \vec{u} = \begin{pmatrix} 4\sqrt{5} \\ 2\sqrt{5} \end{pmatrix}$

Q20. (1) $\begin{pmatrix} -4.5 \\ 6 \end{pmatrix}$

(2) $7.5i + 5j$

Q21. (1) $2\sqrt{13}$

(2) $5\sqrt{5}$

Q22. $\frac{5}{2}\sqrt{10}$

Questions 2.3.

Q23. (1) 93.2° (1.63 rad)
(2) 170° (2.96 rad)
(3) 14° (0.245 rad)
(4) 147° (2.57 rad)
(5) 18.4° (0.322 rad)
(6) 174° (3.03 rad)
(7) 113° (1.98 rad)
(8) 37.2° (0.65 rad)
(9) 135° (2.36 rad)
(10) 156° (2.73 rad)

Q24. 47.9° or 0.837, 20.5

Q25. 60.3° or 1.05

Q26. 59.0° or 1.03

chapter 3

Functions

Questions 3.1.

Q1. (i) none
(ii) injective, surjective, bijective
(iii) surjective
(iv) injective, surjective, bijective
(v) injective

Q2. (a) $a \leq -3$ or $a \geq -2$ (b) $-3 \leq a \leq -2$ (c) $a = -3$ or $a = -2$

Q3. (a) $a \leq -3$ (b) $a \geq -3$ (c) $a = -3$

part II

Algebra

chapter 4

Sequences

4.1 Arithmetic sequences

Questions 4.1.

Q1. 104 **Q2.** −20 **Q3.** 90.5 **Q4.** −10 **Q5.** −3

Q6.

	n	k	a_n	a_k	$\frac{n+k}{2}$	$a_{\frac{n+k}{2}}$
(1)	1	5	7	12	3	9.5
(2)	2	10	3	15	6	9
(3)	6	14	28	3	10	15.5
(4)	13	25	7	11	19	9
(5)	7	15	8	-2	11	3
(6)	12	18	-3	28	15	12.5

$a_{\frac{n+k}{2}} = \frac{a_n + a_k}{2}$

Q7. 620

Q8. 285

Q9. $u_1 = 11, S_{13} = -91$

Q10. (1) 2550
(2) 1225
(3) 2300
(4) 3925
(5) 5040
(6) 3131
(7) 3075
(8) -1767

Q11. 20

Q12. $-3, 2$

Q13. 253

Q14. 385

Q15. 13

Q16. 40 or 125

Q17. 360

Q18. $12.5, 0.25$

Q19. -0.2

Q20. $10, -\frac{1}{3}$

Q21. 80

Q22. 17

Q23. 100

Q24. $3; 168$

Q25. $1; 4, 7$(or $7; 4; 1$)

4.2 Geometric sequences

Questions 4.2.

Q26. $\frac{6561}{64} = 102.515625 \approx 103$

Q27. -1024

Q28. 41984

Q29. 32768

Q30.

	n	k	m	a_n	a_k	a_m	R	S_m
(1)	1	5	8	4	0.25	± 0.03125	± 0.5	$\frac{255}{32} \approx 7.97$ or $\frac{85}{32} \approx 2.66$
(2)	2	6	7	-32	-2	1	-0.5	43
(3)	3	5	8	$\frac{2}{3}$	6	162	3	$\frac{6560}{27}$
(4)	2	3	6	2	3	10.125	1.5	$\frac{665}{24}$
(5)	4	7	9	$-\frac{2}{3}$	18	162	-3	$\frac{9842}{81}$
(6)	1	6	8	$\pm\frac{32}{81}$	96	864	± 3	$\frac{104960}{81}$ or $\frac{52480}{81}$
(7)	2	5	10	-6	48	-1536	-2	-1023

Q31. $0.5; 3$ or $-1; -3$

Q32. $\frac{2}{7}; 2$

Q33. $u_1 = 2, r = 3$ or $u_1 = -4, r = -3$

Q34. $r = \frac{2}{3}, u_1 = 6$

Q35. $u_1 = 2, r = \pm\frac{3}{2}$

Q36. (1) $a = \frac{x+y}{2}$
(2) $a = \pm\sqrt{xy}$

Q37. (i) 927 PLN
(ii) after 62 years

Q38. (i) 929 PLN
(ii) after 61 years and 2 months

Q39. (i) A: 290 PLN, B:118 PLN
(ii) in the 24^{th} month
(iii) in the 35^{th} month

Q40. $x = 6, y = 4$ or $x = \frac{8}{3}, y = -\frac{8}{3}$

Q41. $r = -\frac{2}{3}, u_1 = 3$

Chapter 5

Binomial theorem

Questions 5.1.

Q1. (1) 6 (2) 24 (3) 120 (4) 720 (5) 5040

Q2. (1) 7 (2) 8 (3) 12 (4) 110 (5) 380

Q3. (1) 7 (2) 5 (3) 8

Q4. (1) 3
(2) 4
(3) 6
(4) 10
(5) 10
(6) 5
(7) 6
(8) 15
(9) 20
(10) 15
(11) 21
(12) 35
(13) 35
(14) 56
(15) 28

Q5. (1) $\frac{n(n+1)}{2}$ (3) $\frac{n(n-1)}{2}$ (5) $\frac{2n(2n-1)}{2}$
(2) $\frac{3n(3n-1)}{2}$ (4) $\frac{n(n+1)}{2}$ (6) $\frac{(n+2)(n+1)n}{6}$

Q6. (1) $x^4+4x^3+6x^2+4x+1$
(2) $x^4-4x^3+6x^2-4x+1$
(3) $x^5+10x^4+40x^3+80x^2+80x+32$
(4) $32x^5-80x^4+80x^3-40x^2+10x-1$
(5) $32a^5-80a^4b+80a^3b^2-40a^2b^3+10ab^4-b^5$
(6) $a^8-12a^6b+54a^4b^2-108a^2b^3+81b^4$
(7) $x^6-6x^5y+12x^4y^2-8x^3y^3$
(8) $49-20\sqrt{6}$
(9) $2\sqrt{2}a^3-6a^2b+3\sqrt{2}ab^2-b^3$

Q7. 80.

Q8. -945.

Q9. 160.

Q10. 5376.

Q11. $8-52x+102x^2+...$

Q12. $16+208x+1126x^2+...$

Q13. -2

Q14. *Hint:* $2^n=(1+1)^n$.

CHAPTER 6

Logarithms and exponents

6.1 Algebra of logarithms

Questions 6.1.

Q1. (1) 3 (2) $\frac{1}{4}$ (3) $\frac{\sqrt{2}}{2}$ (4) $\frac{25}{2}$ (5) $4^6=2^{12}$ (6) 11×3^n

Q2. (1) 3 (6) $-\frac{1}{2}$ (11) 2.5 (16) $-\frac{3}{2}$ (21) $\frac{3}{4}$
(2) $\frac{3}{2}$ (7) $-\frac{3}{2}$ (12) $\frac{5}{4}$ (17) $\frac{3}{5}$ (22) $-\frac{7}{4}$
(3) $\frac{2}{3}$ (8) -3 (13) -5 (18) $\frac{1}{2}$ (23) $\frac{1}{12500}$
(4) $\frac{3}{2}$ (9) $-\frac{7}{4}$ (14) 5 (19) $-\frac{8}{3}$ (24) $\frac{2}{\sqrt{3}}$
(5) $\frac{1}{4}$ (10) 3.5 (15) -2 (20) $\frac{5}{6}$ (25) $2\times 3^{\frac{1}{4}}$

Q3. (1) $2b$ (3) $2a+b$ (5) $2+a+b$ (7) $3a$
(2) $a+b$ (4) $3+b$ (6) $3+a+3b$ (8) $3+2a$

Q4. (1) $a+2b$ (3) $\frac{1}{2}a+b$ (5) $a+6b$ (7) $\frac{1}{2}a+4b$
(2) $2a+4b$ (4) $a+2b$ (6) $6b$ (8) a

Q5. (1) $-a$ (3) $\frac{1}{a}$ (5) $\frac{1}{2}a$ (7) $\frac{2+5a}{3}$
(2) $2a$ (4) $1+\frac{3}{a}$ (6) $\frac{1}{3}a$ (8) $\frac{5}{2a}$

Q6. (1) $-a-3b$ (2) $\frac{1}{2}a-b$ (3) $a-\frac{1}{2}b$ (4) $1-4a-\frac{3}{2}b$

Q7. (1) $a-2b-\frac{1}{2}c$ (2) $3a+\frac{1}{2}b-2c)$ (3) $2.5a-1.5b-c$ (4) $-4a-2.5b-\frac{4}{3}c$

Q8. $a=2, b=2$

Q9. $a=2, b=1$

Q10. $a=4, b=1$

Q11. $a=3, b=3$

Q12. $a=3, b=1$

Q13. $a=4, b=2$

Q14. (1) $\frac{3}{4}(2a-1)$ (2) $\frac{2a+3}{4a-2}$ (3) $\frac{3}{2a-1}$ (4) $\frac{14-8a}{3a-3}$

Q15.

(1) $\sqrt{3}$ (3) 8 (5) $2\sqrt{2}$ (7) $\frac{1}{4}$
(2) 25 (4) $3^{(\frac{4}{3})}$ (6) $\frac{1}{9}$ (8) 8

Q16. (1) $2a$ (7) a^3 (13) $\frac{1}{16}a^2$ (19) $\frac{256}{a^{12}}$
(2) $\frac{a}{2}$ (8) $\frac{1}{b^2}$ (14) $\frac{1}{c}$ (20) $\frac{b^6}{243}$
(3) $32a$ (9) c^2 (15) $\frac{16}{a}$ (21) $\frac{1}{3}$
(4) $8a^2$ (10) a^4 (16) $\frac{729}{b^2}$ (22) $\frac{25}{c^5}$
(5) $\frac{1}{3}b^3$ (11) $\frac{1}{a^4}$ (17) c^4 (23) ac
(6) a^2 (12) b^3 (18) $\frac{1}{8}a^6$ (24) $\frac{a^2b^4}{18}$

6.2 Logarithmic equations

Questions 6.2.

Q17. (1) $\frac{27}{64}$ (3) 8 (5) $5^5 = 3125$ (7) $\sqrt[3]{2}$
(2) 27 (4) 9 (6) $\sqrt[3]{2}$ (8) $\frac{1}{2}$

Q18. (1) 4 or $\frac{1}{2}$ (4) 3 (8) -1 or 2 (12) 0
(2) 125 or $\sqrt{5}$ (5) $\frac{1}{9}$ or $\sqrt[3]{3}$ (9) 1 (13) $\frac{1}{\sqrt{3}}$ or $\frac{1}{3}$ or 3
(3) $\sqrt[3]{2}$; $\frac{1}{\sqrt[3]{4}}$ (6) no solutions (10) 0 or 1
(7) 0 (11) 0 or $\frac{1}{2}\log_3 2 - \frac{1}{2}$

Q19. (1) 1 or 3 (2) 3 (3) e (4) $e^{\sqrt{1.5}}$ or $e^{-\sqrt{1.5}}$ or e^3

Q20. (1) 128 (2) 100

Q21. (1) $\frac{5}{8}$ or $\frac{5}{4}$ (4) 2 or -0.4 (7) -3 or 1 (9) $\sqrt{10}$ or 0.001
(2) 9 (5) -2 or 1
(3) 0.1 (6) no solutions (8) 4 or $\frac{1}{2\sqrt{2}}$ (10) $\sqrt{2}$ or 8

Q22. a. $t_0 = 6.6$ c. $6.6(0.75^{10}) \approx 0.372$
b. $k = 0.1 \ln 0.75 \approx -0.0288$ d. $\frac{10 \ln 2}{\ln 0.75} = -\frac{10}{\log_2 0.75} \approx 24.1$

Q23. (1) $x < \frac{64}{\pi^3}$ (4) $x \leq -1.25$ (7) $\frac{1}{5} \leq x \leq 5$
(2) $x \geq \frac{1}{8}$ (5) $x \in \mathbb{R}$
(3) no solutions (6) $x \in \mathbb{R}$ (8) $x \geq 2^{\frac{45}{7}}$

Q24. (i) $0 < x < 1$ or $x > 1$
(ii) 1

chapter 7

Proof by mathematical induction

part III

Functions and equations

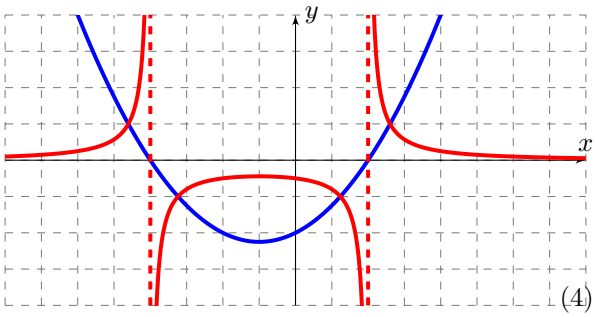
chapter 8

Graphs of rational functions

Questions 8.1.

Q1. Graphs of: $y = f(x)$ and $y = \frac{1}{f(x)}$.

(1)



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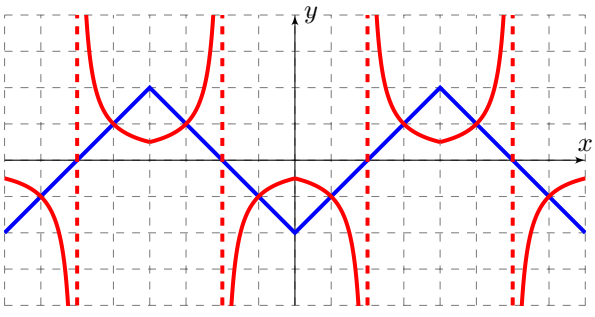

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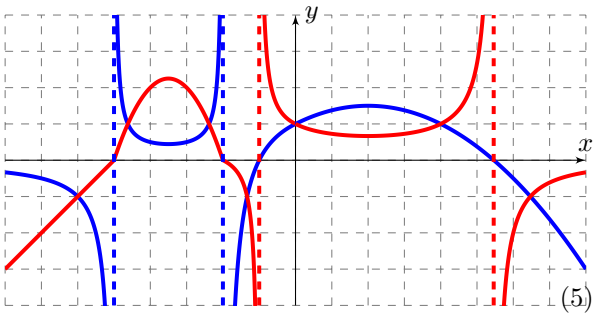

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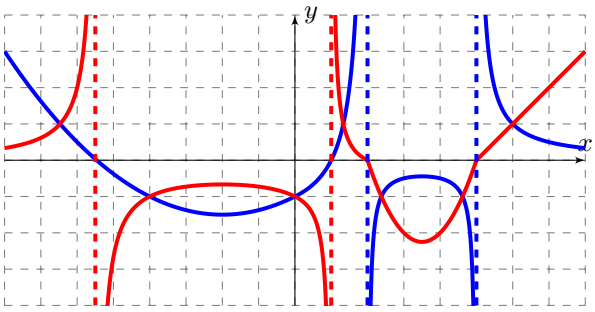

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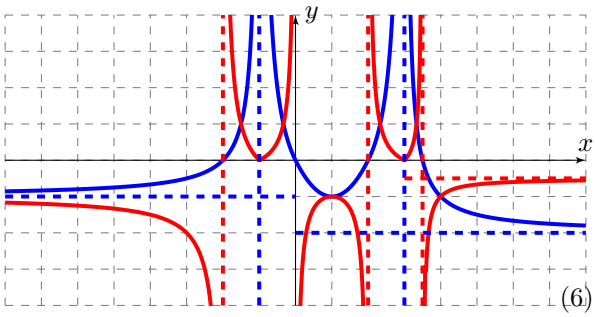

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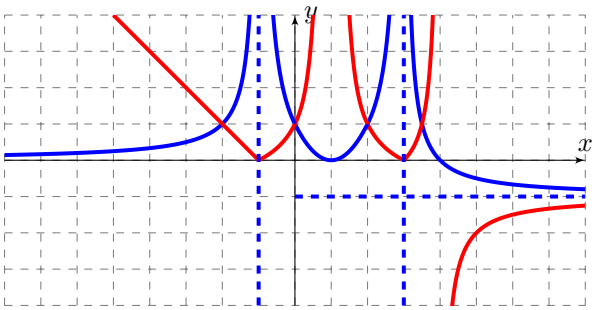

Q2. Graphs of: $x \longmapsto y$ and $x \longmapsto \frac{1}{y}$.

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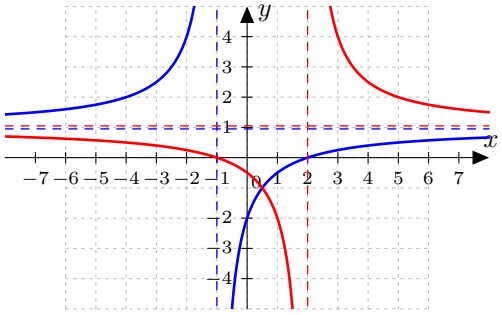

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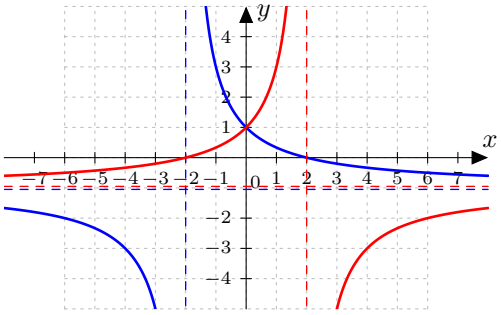

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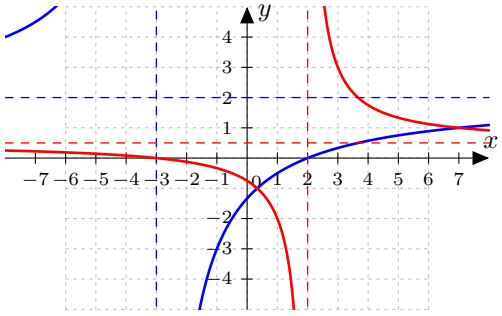

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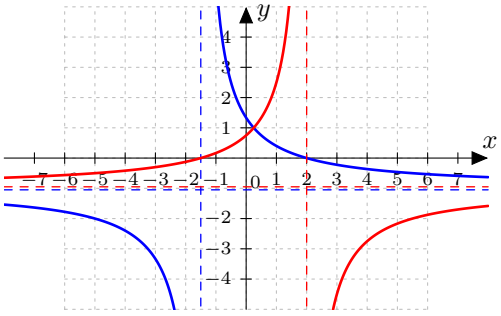

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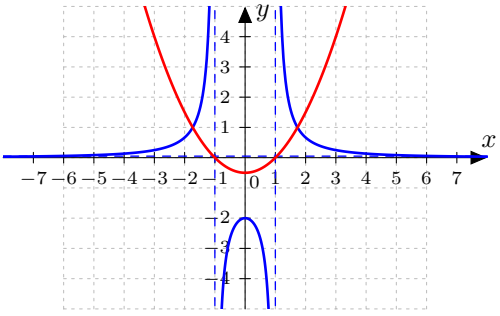

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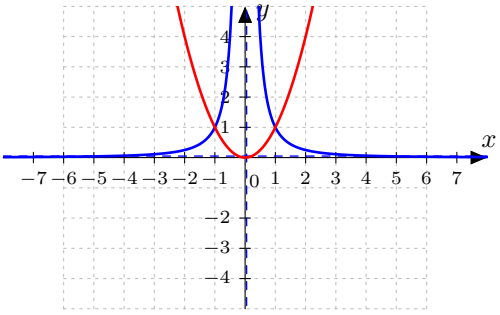

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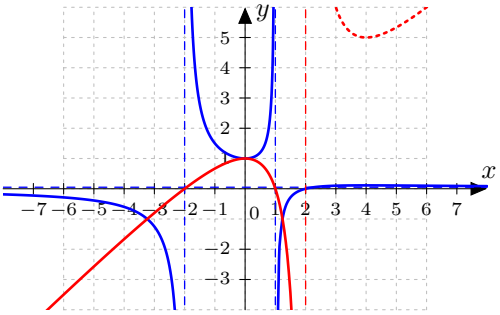

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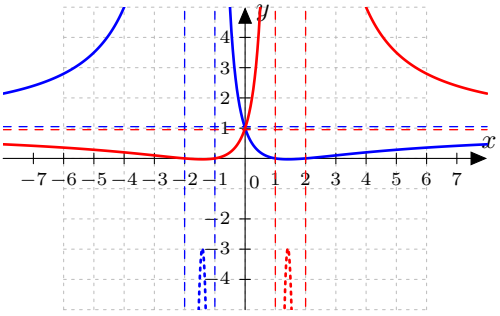

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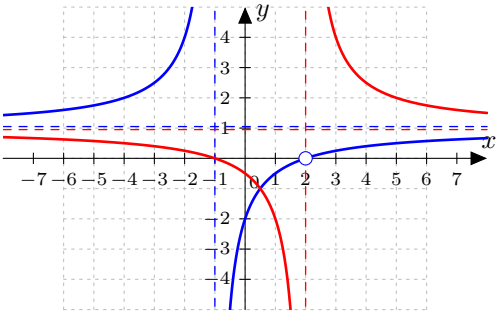

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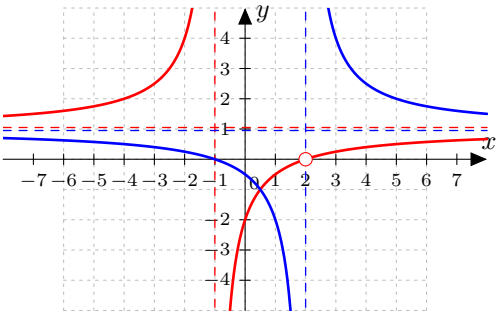

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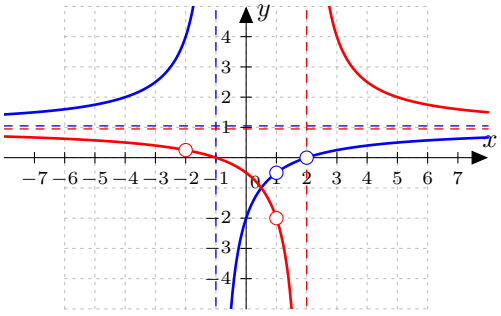

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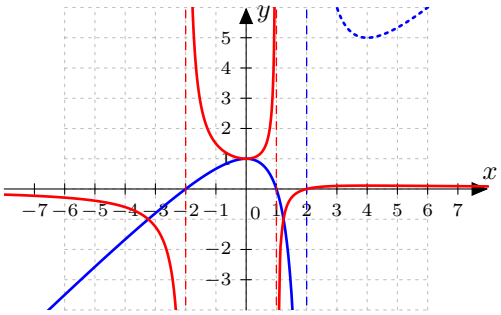

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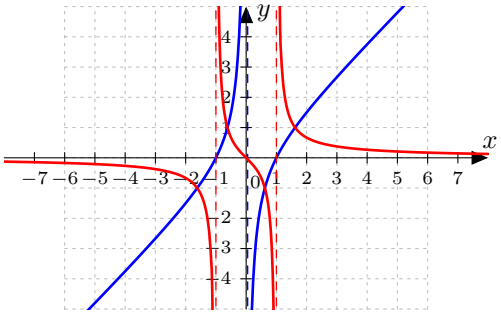

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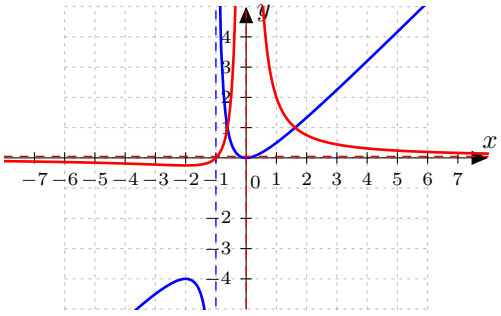

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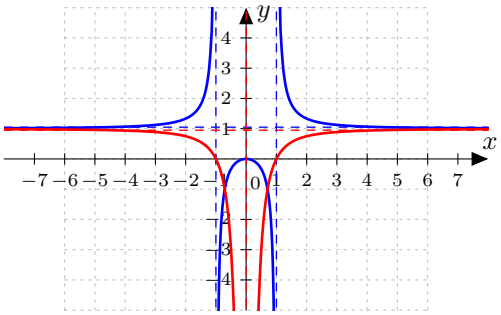


CHAPTER 9

BASIC TRANSFORMATIONS OF GRAPHS OF FUNCTIONS

QUESTIONS 9.1.

Q1. dotted graph→(transformation$_1$, transformation$_2$ ...)→solid graph

(1) $y = x^2$ →(translation by $\begin{pmatrix}-3\\-4\end{pmatrix}$)→ $y = (x+3)^2 - 4$

(2) $y = x^2$ →(vertical dilation by $k = 2$, translation by $\begin{pmatrix}1\\-6\end{pmatrix}$)→ $y = 2(x-1)^2 - 6$

(3) $y = x^2$ →(vertical dilation by $k = -\frac{1}{2}$, translation by $\begin{pmatrix}-2\\3\end{pmatrix}$)→ $y = -\frac{1}{2}(x+2)^2 + 3$

(4) $y = 2^x$ →(translation by $\begin{pmatrix}-3\\-2\end{pmatrix}$)→ $y = 2^{x+3} - 2$

(5) $y = 2^x$ →(reflection in x-axis, translation by $\begin{pmatrix}1\\-2\end{pmatrix}$)→ $y = -2^{x-1} - 2$

(6) $y = \log_{\frac{1}{2}} x$ →(reflection in y-axis, translation by $\begin{pmatrix}-2\\0\end{pmatrix}$)→ $y = \log_{\frac{1}{2}}(-x-2)$

(7) $y = x^2$ →(vertical dilation by $k = 4$, translation by $\begin{pmatrix}2\\-1\end{pmatrix}$)→ $y = 4(x-2)^2 - 1$

(8) $y = \sin x$ →(translation by $\begin{pmatrix}\frac{\pi}{2}\\\end{pmatrix}$)→ $y = \sin(x - \frac{\pi}{2}) + 2$

(9) $y = \sin x$ →(vertical dilation by $k = 2$, translation by $\begin{pmatrix}-\frac{\pi}{3}\\-1\end{pmatrix}$)→ $y = 2\sin(x + \frac{\pi}{3}) - 1$

(10) $y = \sin x$ →(vertical dilation by $k_y = 2$, horizontal by $k_x = -\frac{1}{2}$)→ $y = 2\sin(-2x)$

(11) $y = \sin x$ →(vertical dilation by $k = 4$, translation by $\begin{pmatrix}-\frac{\pi}{2}\\0\end{pmatrix}$)→ $y = 4\sin(x + \frac{\pi}{2})$

(12) $y = \cos x$→(vertical dilation by $k = 3$, translation by $\begin{pmatrix}\frac{\pi}{6}\\0\end{pmatrix}$)→ $y = 3\cos(x - \frac{\pi}{6})$

(13) $y = \cos x$→(vertical dilation by $k = 4$, translation by $\begin{pmatrix}\frac{\pi}{6}\\2\end{pmatrix}$)→ $y = 4\cos(x - \frac{\pi}{6}) + 2$

(14) $y = \cos x$→(vertical dilation by $k = 4$, translation by $\begin{pmatrix}-\frac{\pi}{3}\\-2\end{pmatrix}$)→ $y = 4\cos(x + \frac{\pi}{3}) - 2$

(15) $y = \cos x$→(vertical dilation by $k_y = 4$, horizontal by $k_x = 2$, translation by $\begin{pmatrix}-\frac{\pi}{3}\\0\end{pmatrix}$)→ $y = 4\cos(\frac{1}{2}(x + \frac{\pi}{3}))$

(16) $y = \cos x$→(vertical dilation by $k_y = 5$, horizontal by $k_x = \frac{1}{2}$, translation by $\begin{pmatrix}-\frac{\pi}{4}\\-2\end{pmatrix}$)→ $y = 5\cos(2(x + \frac{\pi}{4})) - 2$

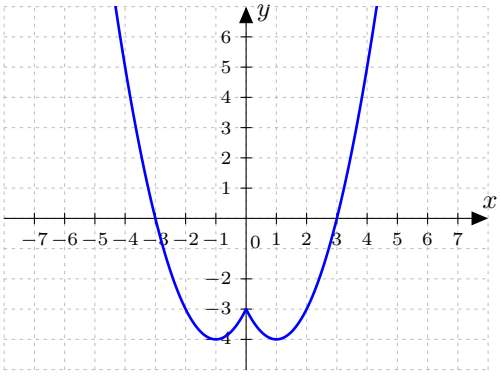
Q2. (1) $y = f(x) \to y = |f(x)|$
(2) reflection in y-axis
(3) horizontal dilation by 2
(4) translation by $\begin{pmatrix}-2\\2\end{pmatrix}$
(5) vertical dilation by $\frac{1}{2}$
(6) reflection in x-axis
(7) $y = f(x) \to y = f(|x|)$

Q3. (1) $\begin{pmatrix}4\\0\end{pmatrix}$ (2) $\begin{pmatrix}2\\0\end{pmatrix}$ (3) $\begin{pmatrix}2\\-2\end{pmatrix}$ (4) $\begin{pmatrix}-2\\0\end{pmatrix}$ (5) $\begin{pmatrix}-\frac{\pi}{2}\\1\end{pmatrix}$ (6) $\begin{pmatrix}\frac{\pi}{4}\\0\end{pmatrix}$ (7) $\begin{pmatrix}-2\\0\end{pmatrix}$ (8) $\begin{pmatrix}1\\1\end{pmatrix}$

Q4. (1) 2 (2) -2 (3) 3 (4) $-\frac{1}{4}$ (5) -2

Q5. (1) $\frac{1}{2}$ (2) 4 (3) 3 (4) -2 (5) $-\frac{3}{2}$ (6) $-\frac{2}{3}$ (7) 4 (8) $\frac{1}{2}$

Q6. (1)



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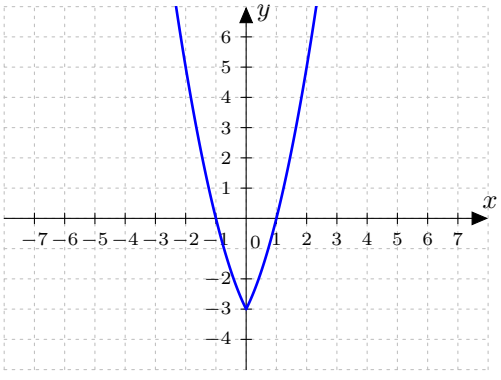

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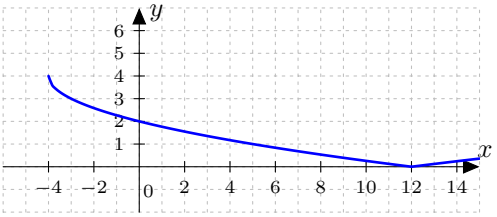

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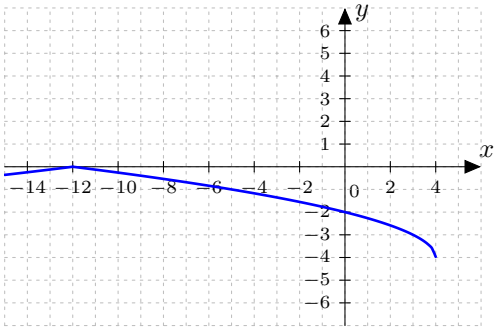

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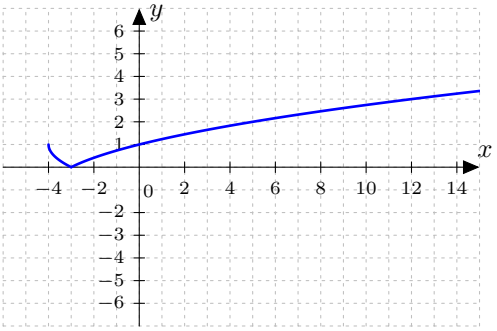

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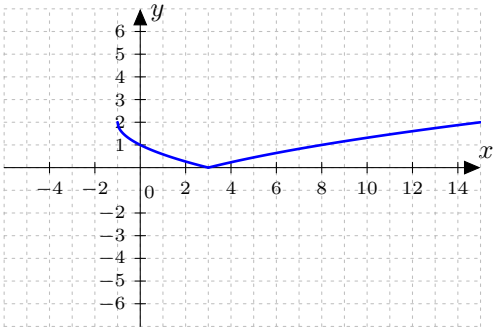

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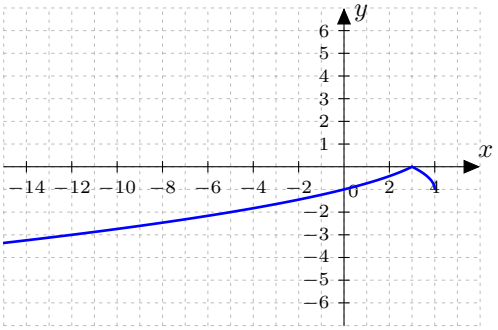

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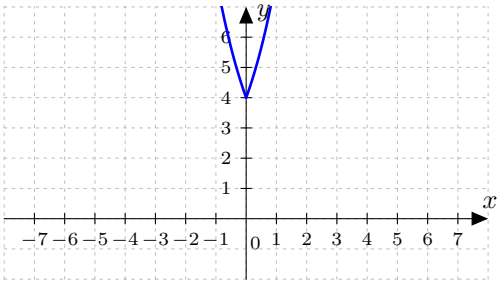

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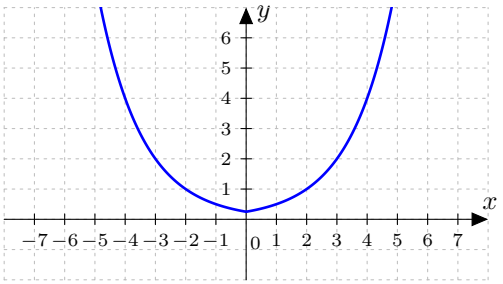

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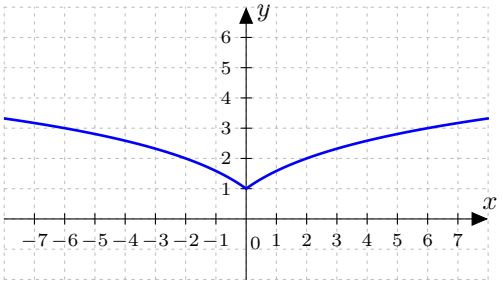
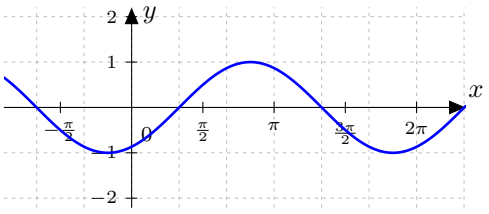


Q7. (1)



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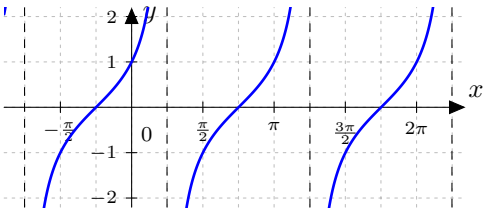

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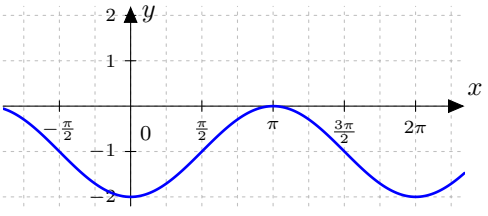

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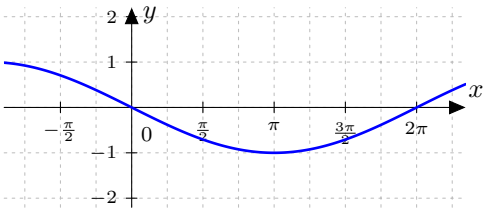

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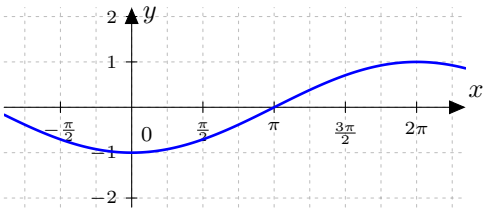

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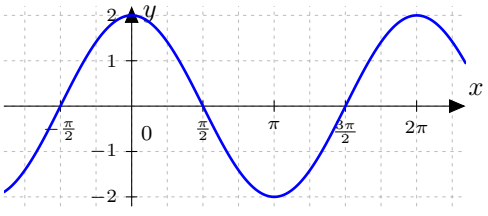

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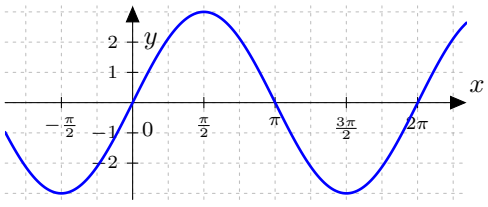

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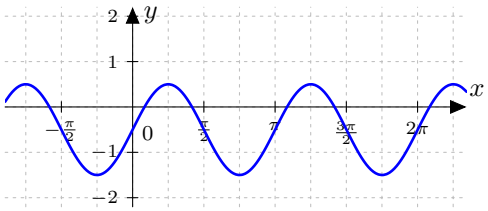

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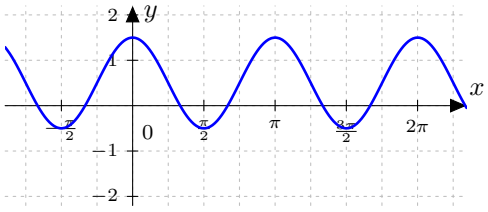

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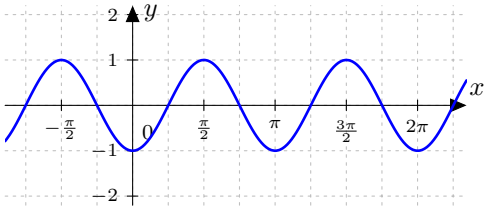

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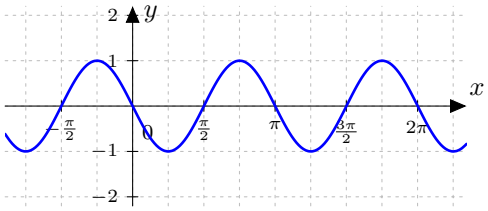

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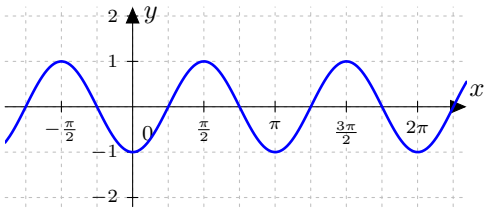

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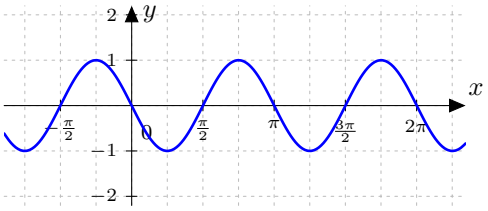

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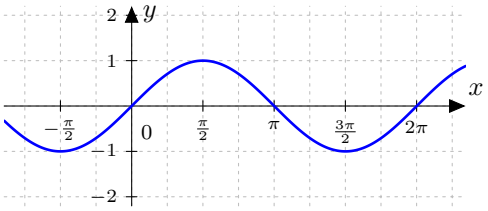

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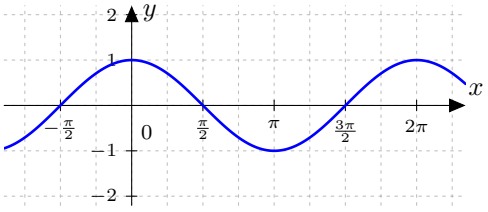

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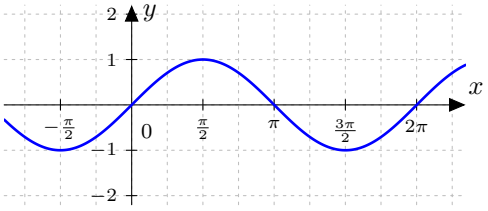

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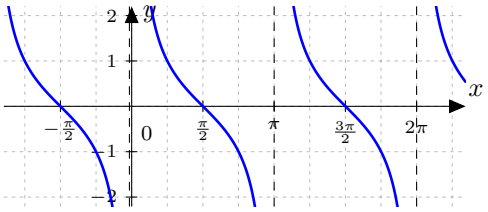

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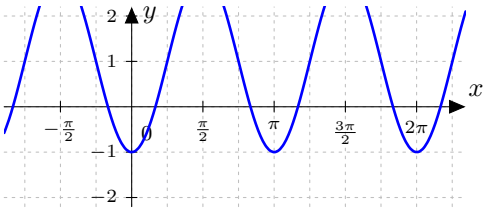


Q8. (exemplary answers)

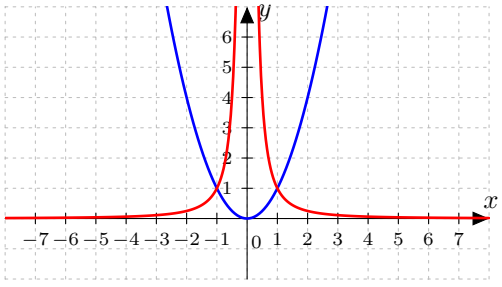
(1) reflection in x-axis (or y-axis) followed by translation by $\binom{\pm\frac{\pi}{2}}{0}$

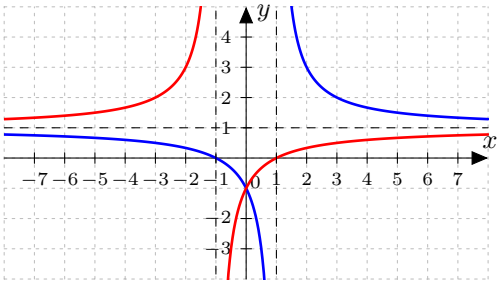
(2) translation by $\binom{\pm\frac{\pi}{2}}{0}$ followed by reflection in x-axis (or y-axis)

(3) translation $\binom{-2}{0}$ followed by reflection in $y = x$ followed by translation $\binom{2}{0}$ (or reflection in $y = x - 2$)

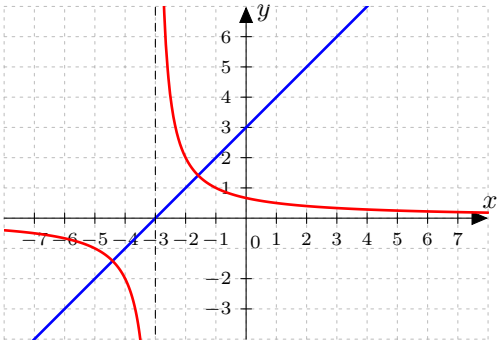
(4) translation $\binom{0}{2}$ followed by reflection in $y = x$ followed by translation $\binom{0}{-2}$ (or reflection in $y = x - 2$)

Q9. Graphs of $x \longmapsto f(x)$ and $x \longmapsto \frac{1}{f(x)}$.

(1)


(2)


(3)



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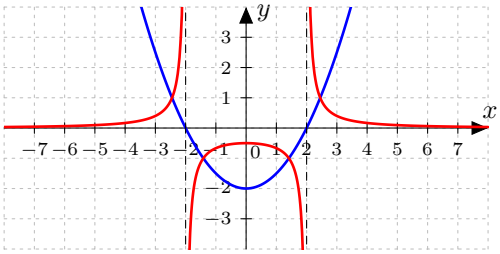

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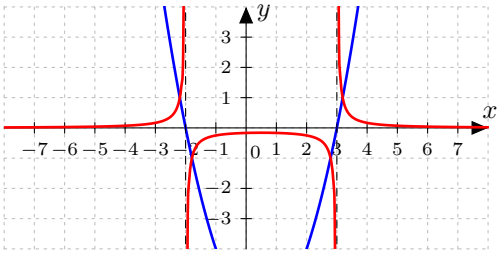

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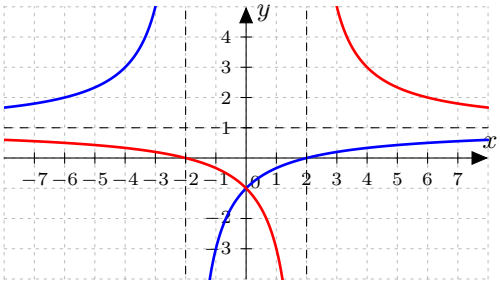

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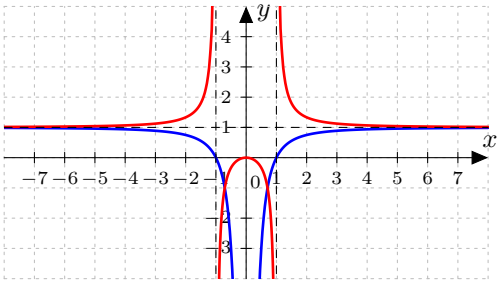

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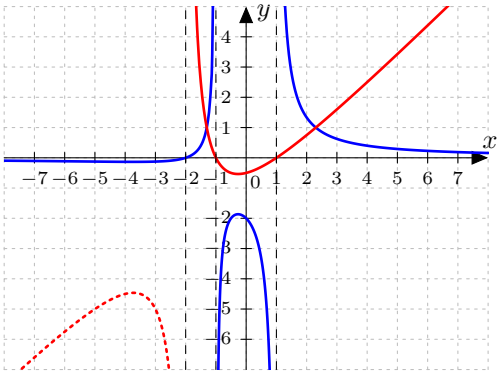

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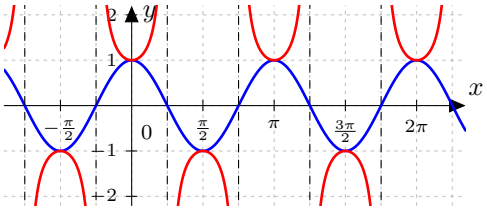

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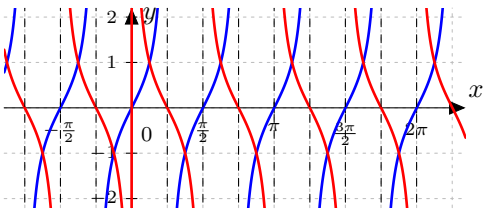
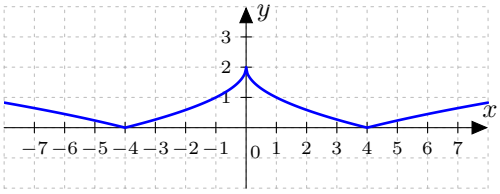
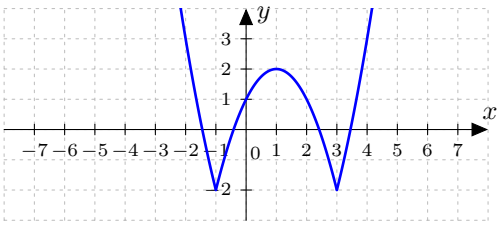


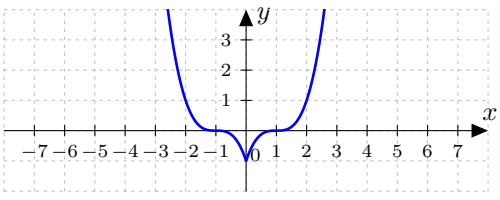
Q10. (1)



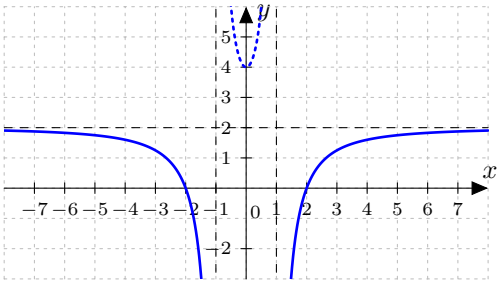
(2)



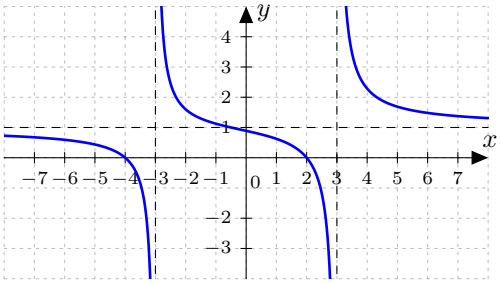
(3)



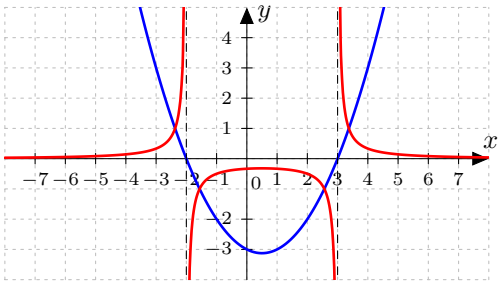
(4)



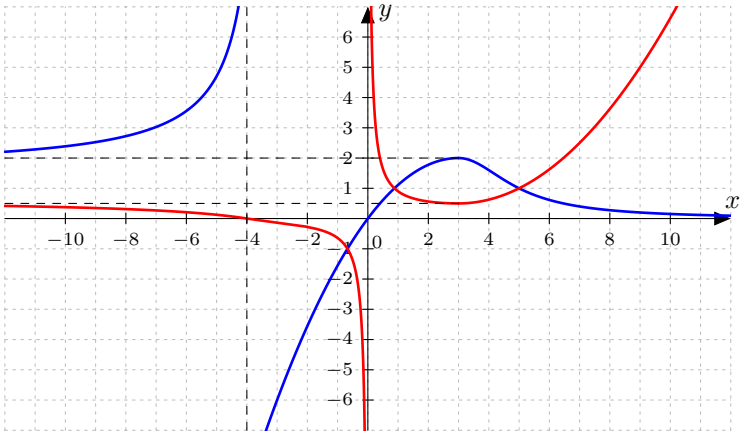
(5)



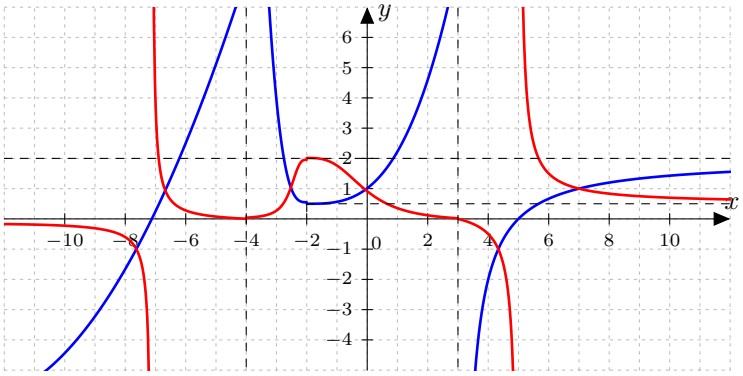
Q11. (1) Graphs of $x \longmapsto f(x)$ and $x \longmapsto \frac{1}{f(x)}$, where $f(x) = \frac{(x-3)(x+2)}{2}$.



(2) Graphs of $x \longmapsto f(x)$ and $x \longmapsto \frac{1}{f(x)}$.

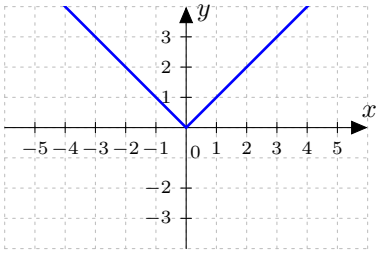


Q12. Graphs of $x \longmapsto f(x)$ and $x \longmapsto \frac{1}{f(x)}$.

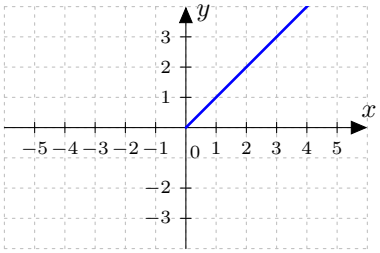


Q13.

(1)

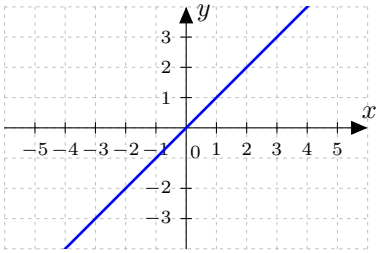


$y = \sqrt{x^2}$

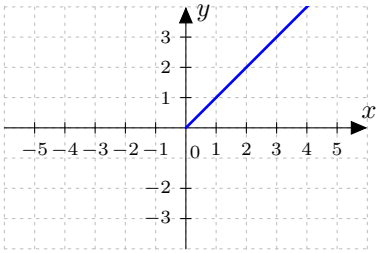


$y = (\sqrt{x})^2$

(2)

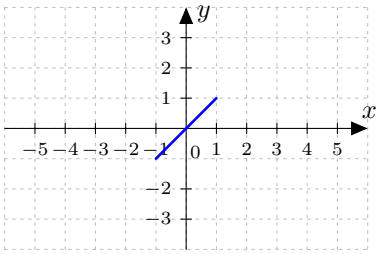


$y = \ln e^x$

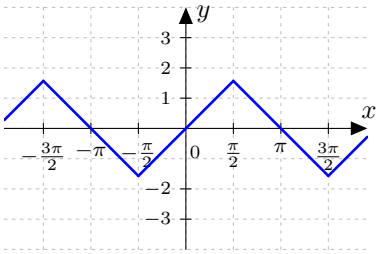


$y = e^{\ln x}$

(3)

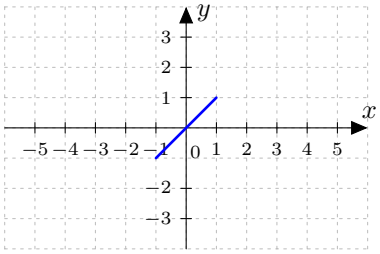


$y = \sin \arcsin x$

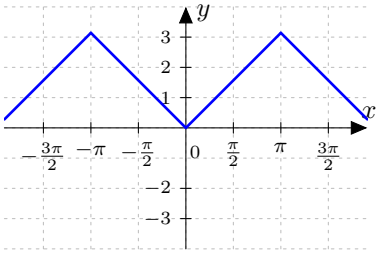


$y = \arcsin \sin x$

(4)

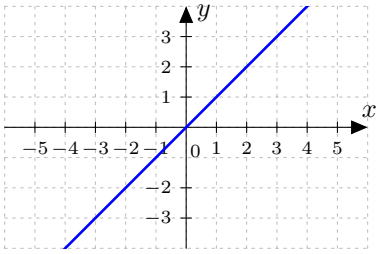


$y = \cos \arccos x$

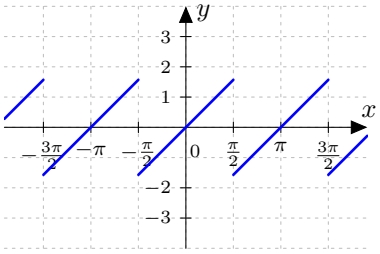


$y = \arccos \cos x$

(5)



$y = \tan \arctan x$



$y = \arctan \tan x$

Q14. $A =]-\infty, 0], B =]0, 1], C =]0, \frac{\pi}{2}]$

Q15. $A =]\frac{\pi}{3}, \frac{2\pi}{3}[, B =]0, 1], C =]-\infty, 0]$

CHAPTER 10

POLYNOMIALS

10.1 ALGEBRA OF POLYNOMIALS

QUESTIONS 10.1.

Q1. (1) $Q(x) = 3x^3 + 2x^2 - x + 3$;
(2) $Q(x) = 4x^3 - 3x^2 + 2$;
(3) $Q(x) = 2x^2 + 5; R = 9$;
(4) $Q(x) = 4x^3 - 2x^2 - x + 2; R = 2$.

Q2. (1) $Q(x) = -2x^4 + x^2 + 2x - 3$;
(2) $Q(x) = x^4 + 3x^3 - 2x^2 + x$;
(3) $Q(x) = -2x^3 + 2x^2 - x + 1$;
(4) $Q(x) = x^3 - 3x^2 + 2x + 0.5$;
(5) $Q(x) = 2x^2 - x + 1; R(x) = x$;
(6) $Q(x) = 3x^2 - \frac{3}{2}; R(x) = \frac{3}{2}x + \frac{1}{2}$.

Q3. (1) $R = 2$;
(2) $R = -58$;.
(3) $R = -60$;
(4) $R = 2$;
(5) $R = 7$;
(6) $R = 4.25$

10.2 REMAINDER THEOREM AND FACTOR THEOREM

QUESTIONS 10.2.

Q4. $R(x) = -2x + 2$

Q5. $R(x) = x$

Q6. $R(x) = -0.2x + 2.4$

Q7. $R(x) = x + 4$

Q8. $R(x) = 3x - 1$

Q9. $R(x) = -2x + 3$

Q10. $a = 2$

Q11. $a = -1; b = 3$

Q12. $a = 3; b = 2$

Q13. $a = -1; b = 5$

Q14. $a = 2; b = 1$

Q15. $a = 4; b = 3$

Q16. $a = 5; b = -5$

Q17. $a = 1; b = 6$

Q18. $a = -1; b = 3$

Q19. $a = 1, b = 2$

Q20. $a = 3, b = -1$

10.3 Vieta's formulas

Questions 10.3.

Q21.
(1) sum$= -2$, product$= 5$
(2) sum$= \frac{1}{2}$, product$= \frac{3}{2}$
(3) sum$= 0$, product$= -2$
(4) sum$= -\frac{3}{4}$, product$= \frac{1}{4}$
(5) sum$= \frac{3}{2}$, product$= 0$
(6) sum$= 0$, product$= -4$
(7) sum$= 0$, product$= 0$
(8) sum$= -2$, product$= -1$

Q22. $a = -6$, product$= \frac{1}{3}$

Q23. $a = 2$, product$= \frac{1}{2}$

Q24. $b = 12$, sum$= \frac{3}{2}$

Q25. $b = -4$, sum$= -3$

Q26. $(0, -6)$

Q27. even

Q28. 4

Q29. $-\frac{1}{2}, 3$

Q30. $\frac{5}{2}, -\frac{3}{2}$

Q31. $a = 4, b = -6$

Q32. $P(x) = x^3 + 5x^2 + 2x - 8$

Q33. solutions: $-1, -2, -4$; $a = 7, b = 14$

Q34. solutions: $1, 5, 9$; $a = -15, b = 59$ or: $-1, -5, -9$; $a = 15, b = 59$

Q35. (1) $4\frac{8}{45}$ (2) $-1\frac{1}{9}$ (3) $-3\frac{13}{27}$ (4) $2\frac{73}{81}$ (5) $7\frac{23}{135}$ (6) $\frac{2}{15}\sqrt{370}$

Q36. (1) $18x^2 + 20x - 15 = 0$ (2) $225x^2 - 940x + 324 = 0$ (3) $27x^2 + 94x + 27 = 0$

10.4 Polynomial inequalities

Questions 10.4.

Q37.
(1) —
(2) $x \leq -4$ or $-\frac{1}{2} \leq x \leq \frac{3}{2}$
(3) $x^2 + x + 1$

Q38.
(1) $1 < x < 2$ or $x > 3$
(2) $x < -4$ or $-1 < x < 2$
(3) $x \leq -5$ or $x = 1$
(4) $x > \frac{3}{2}$
(5) $x < -\frac{1}{2}$
(6) $x \leq -\frac{2}{3}$ or $x \geq 1$
(7) $-2 < x < -\frac{1}{2}$ or $1 < x < 3$
(8) $-2 \leq x \leq 3$
(9) $x \leq -2$ or $x = -\frac{1}{2}$ or $x \geq 3$
(10) $-2 < x < \frac{2}{3}$
(11) $-\frac{1}{2} \leq x \leq \frac{2}{3}$ or $x \geq 2$
(12) $x \leq \frac{1}{3}$ or $x \geq 1$

Q39.
(1) $x \in]-1, 2[\cup]3, +\infty[$
(2) $x \in]-\infty, -4] \cup \{2\}$
(3) $x \in]-\infty, -5[\cup]1, 3[$
(4) $x \in [-2, -1] \cup [4, +\infty[$
(5) $x \in]-\infty, -3[\cup]-2, 3[$
(6) $x \in]-\infty, -2] \cup [3, 5]$
(7) $x \in]-1, 3[\cup]5, +\infty[$
(8) $x \in \{-2\} \cup [2, +\infty[$
(9) $x \in]-1, 1[\cup]2, +\infty[$
(10) $x \in [-4, 2] \cup [3, +\infty[$
(11) $x \in]-\infty, -5[\cup]-1, 3[$
(12) $x \in \{-\frac{7}{3}\} \cup [\frac{3}{2}, +\infty[$
(13) $x \in]-\frac{5}{2}, \frac{5}{3}[\cup]\frac{5}{3}, +\infty[$
(14) $x \in]-\infty, -2] \cup [-1, 2] \cup [3, +\infty[$
(15) $x \in]-\infty, -4] \cup [-1, 1] \cup [2, +\infty[$
(16) $x \in [-5, 1] \cup \{3\}$
(17) $x \in]-1, 4[$
(18) $x \in]-\infty, -3] \cup [-2, 2] \cup [3, +\infty[$
(19) $x \in]2, 3[$
(20) $x \in]-\infty, -2] \cup [0.25, +\infty[$
(21) $x \in]-\infty, -2] \cup [5, +\infty[$
(22) $x \in [-2, -1] \cup [3, 5]$
(23) $x \in]-3, -\frac{1}{2}[\cup]\frac{1}{3}, 2[$
(24) $x \in]-\infty, -3] \cup [-0.5, 1.5] \cup [2, +\infty[$

(25) $x \in]-\infty, -2[\cup]-2, 2[\cup]3, +\infty[$
(26) $x \in]-\infty, 0] \cup \{1.5\} \cup [4, +\infty[$
(27) $x \in]-\frac{1}{2}, \frac{1}{2}[$
(28) $x \in]-\infty, -3] \cup [\frac{1}{2}, +\infty[$
(29) $x \in \emptyset$
(30) $x \in]-\infty, -\frac{5}{2}] \cup [\frac{2}{3}, +\infty[$
(31) $x \in]-\frac{5}{2}, \frac{4}{3}[\cup]\frac{4}{3}, \frac{5}{2}[$

CHAPTER 11

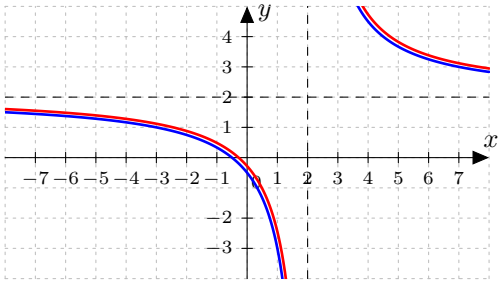
EXPONENTIAL AND LOGARITHMIC FUNCTION

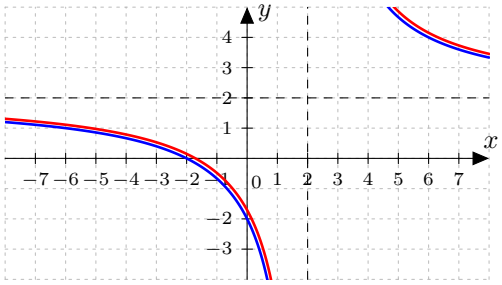
CHAPTER 12

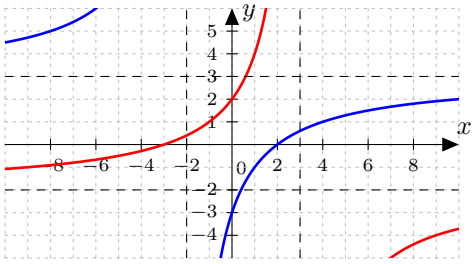
COMPOSITIONS AND INVERSES

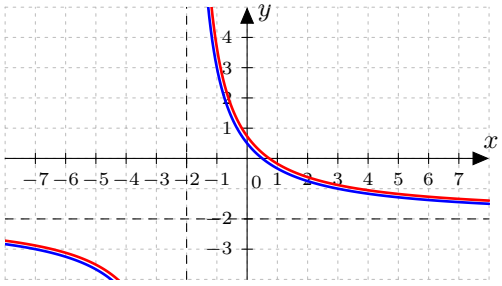
QUESTIONS 12.1.

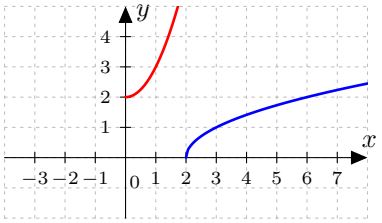
Q1. Graphs of $x \mapsto f(x)$ and $x \mapsto f^{-1}(x)$. D - domain of f / range of f^{-1}, D^{-1} - domain of f^{-1} / range of f.

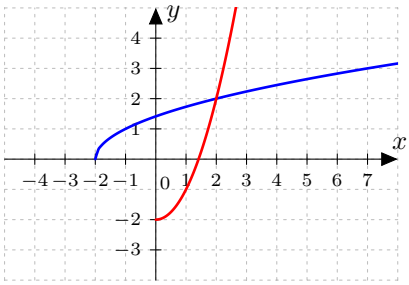
(1)

$f^{-1}(x) = f(x)$, $D = D^{-1} = \mathbb{R} - \{2\}$

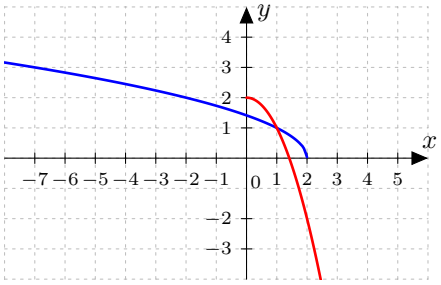
(2)

$f^{-1}(x) = f(x)$, $D = D^{-1} = \mathbb{R} - \{2\}$

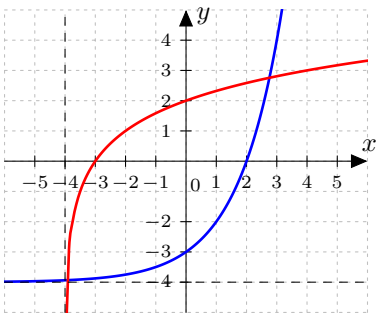
(3)

$f^{-1}(x) = \frac{-2x-6}{x-3}$, $D = \mathbb{R} - \{-2\}, D^{-1} = \mathbb{R} - \{3\}$

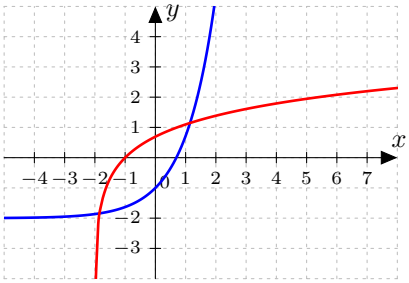
(4)

$f^{-1}(x) = f(x)$, $D = D^{-1} = \mathbb{R} - \{-2\}$

(5)

$f^{-1}(x) = x^2 + 2$, $D = [2, +\infty[, D^{-1} = [0, +\infty[$

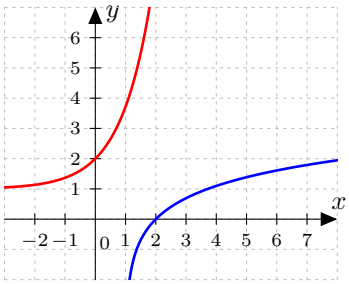
(6)

$f^{-1}(x) = x^2 - 2$, $D = [-2, +\infty[, D^{-1} = [0, +\infty[$

(7)

$f^{-1}(x) = 2 - x^2$, $D =]-\infty, 2[, D^{-1} = [0, +\infty[$

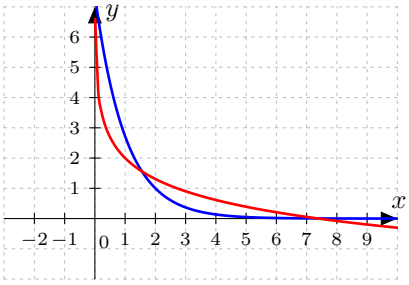
(8)

$f^{-1}(x) = \log_2(x + 4)$, $D = \mathbb{R}, D^{-1} =]-4, +\infty[$

(9) 

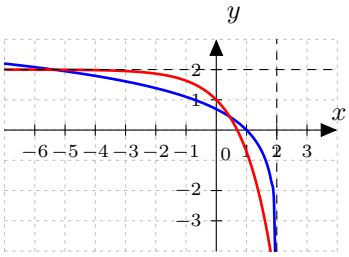
$f^{-1}(x) = \ln(x+2),\ D = \mathbb{R}, D^{-1} =]-2, +\infty[$

(11) 

$f^{-1}(x) = e^x + 1,\ D =]0, +\infty[, D^{-1} = \mathbb{R}$

(10) 

$f^{-1}(x) = 2 - \ln x,\ D = \mathbb{R}, D^{-1} =]0, +\infty[$

(12) 

$f^{-1}(x) = 2 - e^x,\ D =]-\infty, 2[, D^{-1} = \mathbb{R}$

Q2. *(check the graphs with an approprite software)*

(1) (i) $\frac{x+1}{x+2}$ (ii) $x \neq -2,\ y \neq 1$

(2) (i) $\frac{2x+6}{x-3}$ (ii) $x \neq 3,\ y \neq 2$

(3) (i) $\frac{x+2}{-2x+1}$ (ii) $x \neq \frac{1}{2},\ y \neq -\frac{1}{2}$

(4) (i) $\frac{2x+1}{x+1}$ (ii) $x \neq -1,\ y \neq 2$

(5) (i) $\frac{1}{2}\ln(x-1)$ (ii) $x > 1,\ y \in \mathbb{R}$

(6) (i) $1 - \ln(2-x)$ (ii) $x < 2,\ y \in \mathbb{R}$

(7) (i) $3 - e^{x-1}$ (ii) $x \in \mathbb{R},\ y < 3$

(8) (i) $2 - 2^{1-x}$ (ii) $x \in \mathbb{R},\ y < 2$

(9) (i) $3 - (x-2)^2$ (ii) $x \geq 2,\ y \leq 3$

(10) (i) $(x-2)^2 - 1$ (ii) $x \leq 2,\ y \geq -1$

(11) (i) $\sqrt{-x+4} - 1$ (ii) $x \geq -4,\ y \geq -1$

(12) (i) $-\sqrt{2x+4} + 2$ (ii) $x \geq -2,\ y \leq 2$

Q3. (1) $(f \circ g)(x) = 2x^2 + 5$; $(g \circ f)(x) = 4x^2 + 12x + 10$

(2) $(f \circ g)(x) = \sqrt{x^2 + 2x}$; $(g \circ f)(x) = x + 2\sqrt{x-1}$

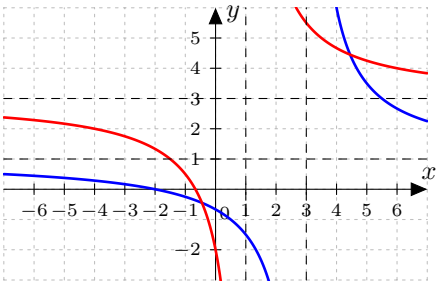
(3) $(f \circ g)(x) = \sqrt[3]{x}$; $(g \circ f)(x) = \sqrt[3]{x-2} + 2$

Q4.

(a) $(f \circ g)(x) = \frac{4x-1}{-x-6}, x \neq -6, x \neq -1$

(b) $f^{-1}(x) = \frac{3x+2}{x-1}$

(c) $x \longmapsto f(x)$, $x \longmapsto f^{-1}(x)$



(d) $x \in]-\infty, -2[\cup]3, +\infty[, y \in \mathbb{R}^*$

(e) $h^{-1}(x) = \frac{3e^x+2}{e^x-1}$.

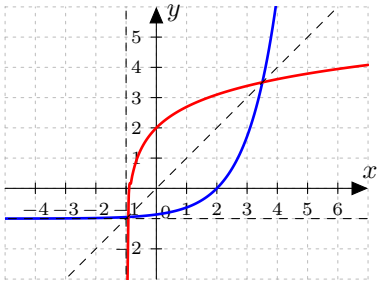
(f) —

(g) $x \neq 1,\ y \in]0, e^2[\cup]e^2, +\infty[$

(h) $k^{-1}(x) = \frac{\ln x + 3}{2 - \ln x}$.

(i) —

Q5. (a) $f^{-1} = \ln(x+1) + 2, x > -1, y \in \mathbb{R}$



(b) $x \longmapsto f(x)$, $x \longmapsto f^{-1}(x)$

(c) $g(x) = \ln(x+4) + 2$

(d) $h(x) = 1 - \ln(x+1)$

Q6. $A = [0, 4[,\ B =]0, 4],\ C =]-\infty, 2]$

Q7. $A =]0, e^3],\ B =]-\infty, 3],\ C = [0, +\infty[$

Q8. $A = [-1, 1], B = [-\frac{\pi}{2}, \frac{\pi}{2}], C = [e^{\frac{\pi}{2}}, e^{-\frac{\pi}{2}}]$

Q9. $A =]-\infty, 0], B =]0, 1], C =]0, \frac{\pi}{2}]$

Q10. $A = \bigcup_{k\in\mathbb{Z}}]\frac{(2k-1)\pi}{3}, \frac{2k\pi}{3}[,\ B =]0, 2], C =]-\infty, \ln 2]$

Q11. $x > 0,\ x \neq 1$, product $= 1$, sum $= \frac{11\sqrt{10}}{10}$

Q12. domain:

(i) $x \leq 1$ (ii) $\mathbb{R}$ (iii) $x \leq 0$ (iv) $x \leq 1$

range:

(i) $y \geq 0$ (ii) $y > 0$ (iii) $0 \leq y < 1$ (iv) $y \geq 1$

Q13. (i) $a = 2$ (ii) $y \geq -1$ (iii) $f^{-1}(x) = \sqrt{x+1} + 2$

Q14. (1) $f^{-1}(x) = \sqrt{\ln(x+2)},\ x \geq -1\ y \geq 0$

(2) $f^{-1}(x) = -\sqrt{\ln(x-1)},\ x \geq 2,\ y \leq 0$

(3) $f^{-1}(x) = -\sqrt{e^{x+1} - 1},\ x \geq -1,\ y \leq 0$

(4) $f^{-1}(x) = \sqrt{1 - e^x},\ x \leq 0,\ 0 \leq y < 1$

(5) $f^{-1}(x) = 3\sqrt{4 - x^2},\ 0 \leq x \leq 2,\ 0 \leq y \leq 6$

(6) $f^{-1}(x) = -\sqrt{x^2 + 1},\ x \geq 0,\ y \leq -1$

Q15. $f^{-1}(x, y) = (\frac{2x+y}{3}, \frac{x-y}{3})$

CHAPTER 13

INEQUALITIES

QUESTIONS 13.1.

Q1. (1) $0 < x < 1$ or $x > 3$

(2) $-\frac{3}{2} \leq x < 2$

(3) $x < 0$ or $1 < x < 2$

(4) $\frac{5}{3} < x < 2$

(5) $1 < x < 2$

(6) $-1 < x \leq 4$

(7) $x > 1$

(8) $x < -1$ or $\frac{1}{2} < x < 2$

(9) $-2 < x < \frac{3}{2}$

(10) $-1 < x \leq 0$ or $\frac{1}{2} < x \leq 2$

(11) $x < -4$ or $0 \leq x < 4$ or $x \geq 16$

(12) $-2 < x < 0$

(13) $-\frac{1}{3} \leq x < \frac{1}{2}$ or $\frac{1}{2} < x \leq 3$

(14) $-\frac{1}{5} \leq x \leq 3$

(15) $-1 < x < 2$

(16) $x < -10$ or $x > 2$

(17) $x \leq \frac{5}{3}$ or $x \geq 3$

Q2. (1) $x < 0.589$ or $3.10 < x < 4$

(2) $0 < x < 0.832$ or $2.61 < x < 3.58$ or $6.06 < x < 6.47$

(3) $2.57 \leq x \leq 7.40$ or $x \geq 8.22$

(4) $x < -1.71$ or $0 < x < 1.71$

PART IV

CIRCULAR FUNCTIONS AND TRIGONOMETRY

CHAPTER 14

CIRCULAR FUNCTIONS

14.1 BASIC CIRCULAR FUNCTIONS

14.2 UNIT CIRCLE

Questions 14.1.

Q1.
(1) $\frac{\sqrt{3}}{2}$
(2) $-\frac{\sqrt{3}}{2}$
(3) 1
(4) none
(5) $-\sqrt{3}$
(6) $-\frac{\sqrt{3}}{2}$
(7) 0
(8) $-\sqrt{3}$
(9) $-\frac{\sqrt{3}}{2}$
(10) $-\frac{1}{2}$
(11) none
(12) $\frac{1}{2}$
(13) -1
(14) 0
(15) $\sqrt{3}$
(16) $-\frac{1}{2}$
(17) $-\frac{1}{2}$
(18) $-\frac{\sqrt{3}}{2}$
(19) $-\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$
(20) $\frac{1}{2}$
(21) $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

Q2.

	$\sin A$	$\cos A$	$\tan A$
(1)	given	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
(2)	given	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$
(3)	given	$\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$
(4)	given	$-\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
(5)	$\frac{3}{5}$	given	$\frac{3}{4}$
(6)	$\frac{2\sqrt{2}}{3}$	given	$-2\sqrt{2}$
(7)	Not possible	Not possible	Not possible
(8)	$-\frac{2\sqrt{2}}{3}$	given	$-2\sqrt{2}$
(9)	$-\frac{4}{5}$	given	$\frac{4}{3}$
(10)	$\frac{3}{5}$	$\frac{4}{5}$	given
(11)	$-\frac{4}{5}$	$-\frac{3}{5}$	given
(12)	$\frac{3}{\sqrt{10}}$	$-\frac{1}{\sqrt{10}}$	given
(13)	$-\frac{3}{\sqrt{10}}$	$-\frac{1}{\sqrt{10}}$	given
(14)	Not possible	Not possible	Not possible

Q3.

	$\sin A$	$\cos A$	$\tan A$
(1)	given $(t \geq 0)$	$\sqrt{1-t^2}$	$\frac{t}{\sqrt{1-t^2}}$
(2)	given $(t \geq 0)$	$-\sqrt{1-t^2}$	$-\frac{t}{\sqrt{1-t^2}}$
(3)	given $(t < 0)$	$\sqrt{1-t^2}$	$\frac{t}{\sqrt{1-t^2}}$
(4)	given $(t < 0)$	$-\sqrt{1-t^2}$	$-\frac{t}{\sqrt{1-t^2}}$
(5)	$\sqrt{1-t^2}$	given $(t \geq 0)$	$\frac{\sqrt{1-t^2}}{t}$
(6)	$\sqrt{1-t^2}$	given $(t < 0)$	$\frac{\sqrt{1-t^2}}{t}$
(7)	$-\sqrt{1-t^2}$	given $(t \geq 0)$	$-\frac{\sqrt{1-t^2}}{t}$
(8)	$-\sqrt{1-t^2}$	given $(t < 0)$	$-\frac{\sqrt{1-t^2}}{t}$
(9)	$\frac{t}{\sqrt{t^2+1}}$	$\frac{1}{\sqrt{t^2+1}}$	given $(t \geq 0)$
(10)	$-\frac{t}{\sqrt{t^2+1}}$	$-\frac{1}{\sqrt{t^2+1}}$	given $(t < 0)$
(11)	$\frac{t}{\sqrt{t^2+1}}$	$\frac{1}{\sqrt{t^2+1}}$	given $(t < 0)$
(12)	$-\frac{t}{\sqrt{t^2+1}}$	$-\frac{1}{\sqrt{t^2+1}}$	given $(t \geq 0)$
(13)	given $(t < 0)$	$-\sqrt{1-4t^2}$	$-\frac{2t}{\sqrt{1-4t^2}}$
(14)	$\sqrt{1-9t^2}$	given $(t \geq 0)$	$\frac{\sqrt{1-9t^2}}{3t}$
(15)	$\frac{1}{2}\sqrt{4-t^2}$	given $(t < 0)$	$\frac{\sqrt{4-t^2}}{t}$
(16)	$\frac{5t}{\sqrt{25t^2+4}}$	$\frac{2}{\sqrt{25t^2+4}}$	given $(t < 0)$

Q4. (All answers given to 3 s.f.)

(1) 0.201
(2) 2.37
(3) -2.94
(4) 2.69
(5) -2.69
(6) -1.47
(7) 2.03
(8) -2.85

Q5. (1) $-\frac{\pi}{3}$ (2) $\frac{5\pi}{6}$ (3) $\frac{\pi}{2}$ (4) $\frac{5\pi}{6}$ (5) $-\frac{3\pi}{4}$ (6) $-\frac{2\pi}{3}$ (7) $\frac{5\pi}{6}$

Q6. (1) $\sqrt{1-q^2}$ (2) $\frac{\sqrt{1-q^2}}{q}$ (3) $-\sqrt{1-p^2}$ (4) $-q$ (5) $\sqrt{1-q^2}$ (6) $\sqrt{1-p^2}$ (7) $\frac{p}{\sqrt{1-p^2}}$

14.3 Graphs of trigonometric functions

Questions 14.2.

Q7. Check your answers with a GDC or at *wolframalpha.com*.

Q8. $A = -5, B = 2, C = 1$

Q9. $p = -4, q = \frac{\pi}{2}, s = 2$

Q10. (i) $a = 3, b = \pi, c = \pi, d = 2$
(ii) $m = -1$.
(iii) $-1 < k < 2$.

Q11. (i) $y = 12\sin(\frac{\pi}{4}x - \frac{5\pi}{4}) + 5$
(ii) $y = 12\cos(\frac{\pi}{4}x + \frac{\pi}{4}) + 5$

Q12. (i) $y = 0.6\sin(\frac{\pi}{8}x + \frac{\pi}{4}) - 0.2$
(ii) $y = 0.6\cos(\frac{\pi}{8}x - \frac{\pi}{4}) - 0.2$
(iii) $y = 0.6\sin(\frac{\pi}{4}x + \frac{9\pi}{40}) + 0.8$

Q13. (i) $y = 70\sin(\frac{3}{2}x - \frac{\pi}{4}) - 30$
(ii) $y = 70\cos(\frac{3}{2}x - \frac{3\pi}{4}) - 30$
(iii) $y = 35\sin(\frac{3}{2}x - \frac{\pi}{2}) + 5$
or $y = 35\cos(\frac{3}{2}x - \pi) + 5$

Q14.

	expression	period	range
$f \circ g(x)$	$\frac{1}{6}\sin(6x) + \frac{2}{3}$	$\frac{\pi}{3}$	$[\frac{1}{2}, \frac{5}{6}]$
$g \circ f(x)$	$\frac{1}{4}\sin(4x) + 1$	$\frac{\pi}{2}$	$[\frac{3}{4}, \frac{5}{4}]$

Q15. (i) $d = 14, t = 8$
(ii) $d = 2, t = 20$
(iii) $d = 6\sin(\frac{\pi}{12}t - \frac{\pi}{6}) + 8$
(iv) $[1.36, 14.6]$ or from 1 : 22AM to 2 : 38PM

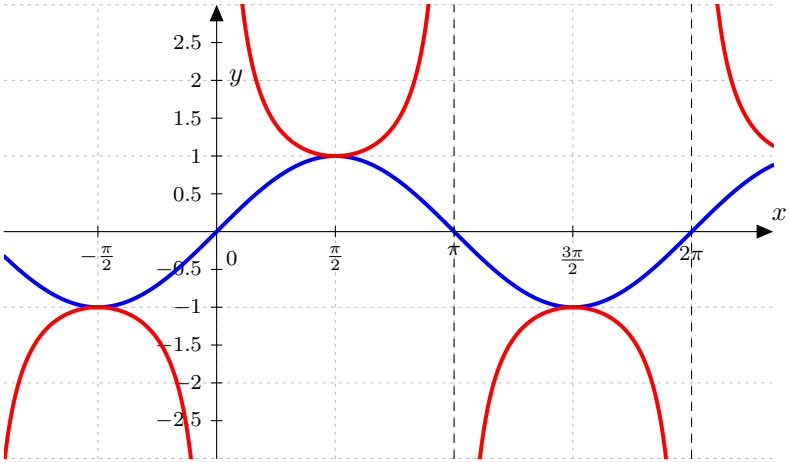
Q16. (i) $A = 5.5, B = \frac{\pi}{6}, C = -\frac{\pi}{6}, D = 12.5$
(ii) 41.2%

Q17. (i) $210cm$
(ii) 2 seconds
(iii) $206cm$
(iv) $t = \frac{7}{6}s \approx 1.17s$

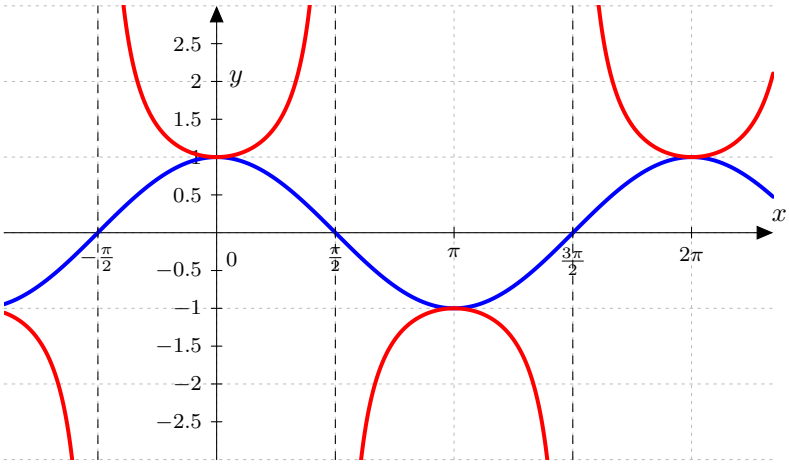
14.4 Cosecant, secant and cotangent - reciprocal circular functions

Questions 14.3.

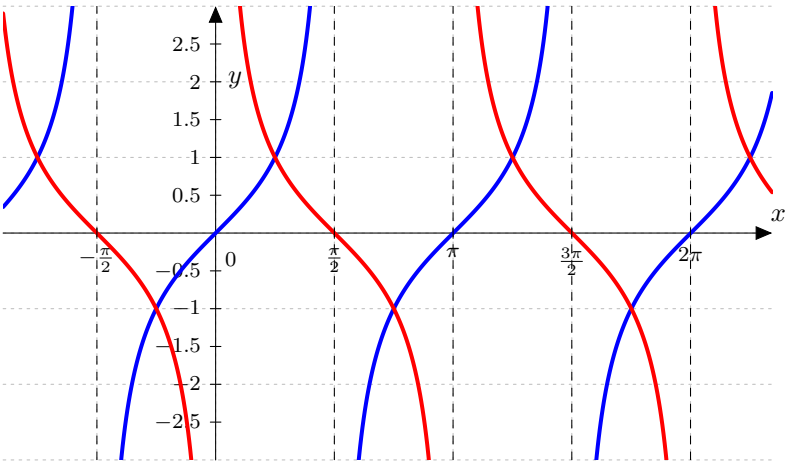
Q18. (1) $x \longmapsto \sin x$ and $x \longmapsto \csc x$



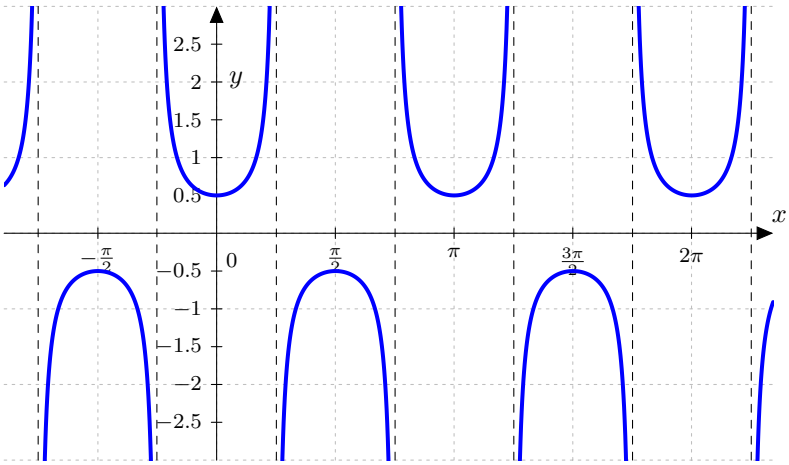
(2) $x \longmapsto \cos x$ and $x \longmapsto \sec x$



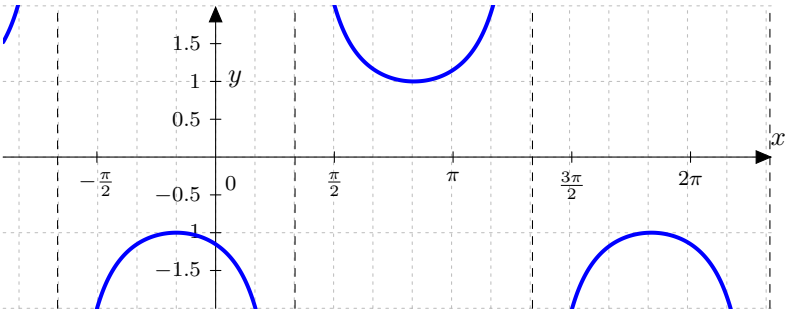
(3) $x \longmapsto \tan x$ and $x \longmapsto \cot x$



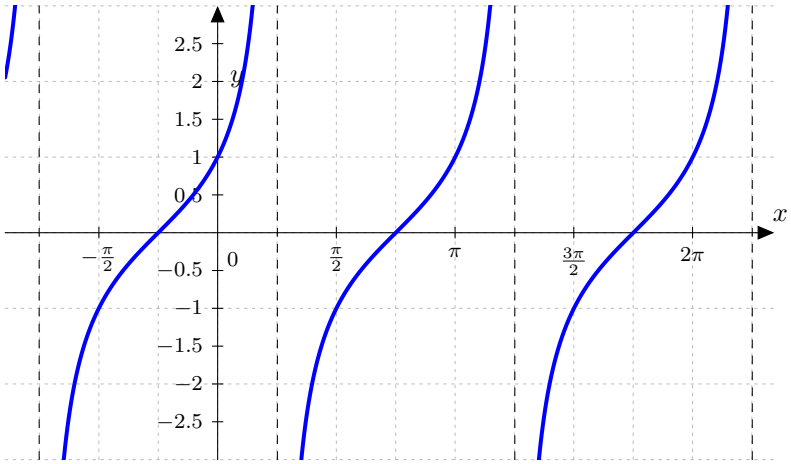
Q19. (1)



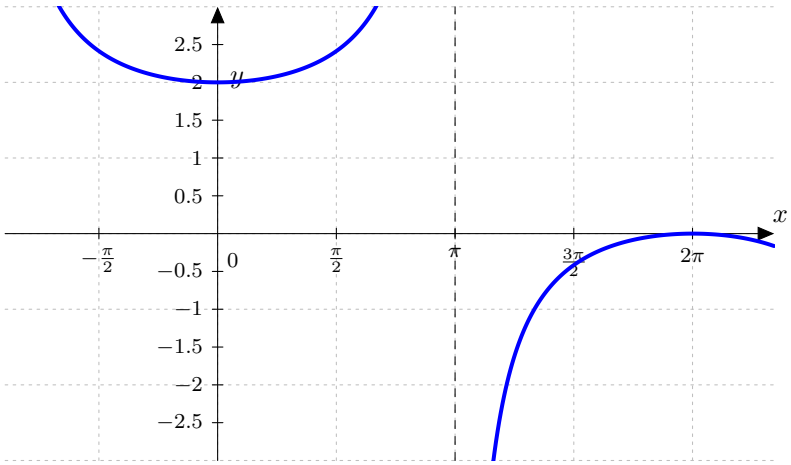
(2)



(3)



(4)



Q20. (1) $-\frac{\pi}{4}$ (2) $-\frac{2\pi}{3}$ (3) $-\frac{\pi}{6}$

Q21. (1) $x \in \{\frac{\pi}{3}, \frac{5\pi}{3}\}$
(2) $x \in \{\frac{\pi}{3}, \frac{2\pi}{3}\}$
(3) $x \in \{\frac{\pi}{6}, \frac{7\pi}{6}\}$
(4) $x = \frac{4\pi}{3}$
(5) $x = \frac{3\pi}{2}$
(6) $x \in \{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\}$

Q22. **hint**: use Pythagoras' theorem to show the first identity and then divide its both sides by a square of sine or cosine.

Q23.

	$\csc x$	$\sec x$	$\cot x$
1	given	$-3/(2\sqrt{2})$	$-2\sqrt{2}$
2	–	–	–
3	$\sqrt{10}$	$-\frac{\sqrt{10}}{3}$	given
4	–	–	–
5	$-\frac{13}{12}$	given	$\frac{5}{12}$
6	given	$\frac{5}{\sqrt{21}}$	$-\frac{\sqrt{21}}{2}$

14.5 Inverse circular functions

Questions 14.4.

Q24.

function	domain	range
$x \mapsto \arcsin x$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$x \mapsto \arccos x$	$[-1, 1]$	$[0, \pi]$
$x \mapsto \arctan x$	$\mathbb{R}$	$]-\frac{\pi}{2}, \frac{\pi}{2}[$

Q25.

	$\sin A$	$\cos A$	$\tan A$
(1)	$\frac{\sqrt{7}}{3}$	$\frac{\sqrt{2}}{3}$	$\frac{\sqrt{14}}{2}$
(2)	$\frac{3}{4}$	$-\frac{\sqrt{7}}{4}$	$-\frac{3}{\sqrt{7}}$
(3)	$\frac{2}{3}$	$\frac{\sqrt{5}}{3}$	$\frac{2}{\sqrt{5}}$
(4)	$-\frac{3}{5}$	$\frac{4}{5}$	$-\frac{3}{4}$
(5)	$\frac{3}{\sqrt{10}}$	$\frac{1}{\sqrt{10}}$	3
(6)	$-\frac{2}{\sqrt{13}}$	$\frac{3}{\sqrt{13}}$	$-\frac{2}{3}$

Q26. (All answers given to 3 s.f.)

	$A \in$	$s = \sin A$	$c = \cos A$	$t = \tan A$	$\arcsin s$	$\arccos c$	$\arctan t$	A
(1)	$[\frac{\pi}{2}, \pi]$	0.768	−0.64	−1.2	0.876	2.27	−0.876	2.27
(2)	$[\frac{\pi}{2}, \pi]$	0.908	−0.42	−2.16	1.14	2	−1.14	2
(3)	$[-\pi, -\frac{\pi}{2}]$	−0.253	−0.967	0.262	−0.256	2.89	0.256	−2.89
(4)	$[-\frac{\pi}{2}, 0]$	−0.419	0.908	−0.461	−0.432	0.432	−0.432	−0.432
(5)	$[-\pi, -\frac{\pi}{2}]$	−0.954	−0.298	3.2	−1.27	1.87	1.268	−1.87
(6)	$[\frac{\pi}{2}, \pi]$	0.196	−0.981	−0.2	0.197	2.94	−0.197	2.94
(7)	$[-\pi, -\frac{\pi}{2}]$	−0.029	−1	0.029	−0.029	3.11	0.029	−3.11

Q27. (1) $\sqrt{1-a^2}$ (2) $\frac{a}{\sqrt{1+a^2}}$ (3) $\sqrt{1-a^2}$ (4) $\frac{1}{\sqrt{1+a^2}}$ (5) $\frac{a}{\sqrt{1-a^2}}$ (6) $\frac{\sqrt{1-a^2}}{a}$ (7) $\frac{a}{\sqrt{1-a^2}}$ (8) $\frac{1}{a}$ (9) $\frac{\sqrt{1-a^2}}{a}$ (10) $\frac{1}{a}$ (11) $\sqrt{1+a^2}$ (12) $\frac{1}{\sqrt{1-a^2}}$ (13) $\frac{1}{\sqrt{1-a^2}}$ (14) $\frac{\sqrt{1+a^2}}{a}$ (15) $\frac{1}{a}$

Q28. (1) $-\frac{1}{2} \le x <\le frac12; -\frac{\pi}{2} \le y \le \frac{\pi}{2}$ (2) $-2 \le x \le 2; -3\pi \le y \le 0$ (3) $x \ge 0; 0 \le y < \sqrt{\frac{\pi}{2}}$ (4) $x \le 1, 0 \le y < \frac{\sqrt{3\pi}}{2}$ (5) $-1 \le x \le 1, x \ne 0; y \le -\frac{2}{\pi}$ or $y \ge \frac{2}{\pi}$ (6) $-1 \le x < 1; y \ge \frac{2}{\pi}$ (7) $-1 \le x \le \frac{1}{2}; 0 \le y \le \sqrt{\frac{2\pi}{3}}$ (8) $\frac{\sqrt{2}}{2} \le x \le 1; 0 \le y \le \sqrt{\frac{\pi}{4}}$ (9) $-1 \le x \le \frac{\sqrt{2}}{2}; 0 \le y \le \sqrt{\frac{3\pi}{4}}$

CHAPTER 15

TRIGONOMETRIC IDENTITIES

QUESTIONS 15.1.

Q1. $\sin(\frac{\pi}{8}) = \frac{\sqrt{2-\sqrt{2}}}{2}$, $\cos(\frac{5\pi}{12}) = \frac{\sqrt{6}-\sqrt{2}}{4}$, $\tan(\frac{\pi}{12}) = 2 - \sqrt{3}$.

Q2. $\frac{\sqrt{3}-2\sqrt{2}}{6}$ **Q3.** $-\frac{\sqrt{15}+2}{6}$ **Q4.** -7 **Q5.** — **Q6.** — **Q7.** — **Q8.** —

Q9. (1) 1 (2) 1 (3) $\frac{1}{5}$ (4) $\frac{1}{2}$ (5) $\frac{1}{50}$ (6) $\frac{19}{9}$ (7) $\frac{1}{3}$ (8) 1 (9) $\frac{7}{40}$ (10) $\frac{1}{2}$

Q10. (i) $R = \sqrt{5}, \alpha \approx 0.464$ (ii) $\min = -2\sqrt{5}$, $\max = 4\sqrt{5}$ (iii) $\min = \frac{2}{3+\sqrt{5}}$, $\max = \frac{2}{3-\sqrt{5}}$

CHAPTER 16

TRIGONOMETRIC EQUATIONS AND INEQUALITIES

QUESTIONS 16.1.

Q1. (1) $x \in \{\frac{\pi}{6}, \frac{5\pi}{6}\}$
(2) $x \in \{-\frac{11\pi}{6}, -\frac{7\pi}{6}\}$
(3) $x \in \{-\frac{5\pi}{6}, -\frac{\pi}{6}\}$
(4) $x \in \{\frac{7\pi}{6}, \frac{11\pi}{6}\}$
(5) $x \in \{-\frac{\pi}{2}, \frac{3\pi}{2}\}$
(6) $x \in \{0.412, 2.73\}$
(7) $x \in \{3.55, 5.87\}$
(8) $x \in \{-5.87, -3.55\}$
(9) $x \in \{-2.73, -0.412\}$
(10) $x \in \{-\frac{\pi}{2}, \frac{3\pi}{2}\}$

Q2. (1) $x \in \{\frac{\pi}{3}, \frac{5\pi}{3}\}$
(2) $x \in \{-\frac{5\pi}{3}, -\frac{\pi}{3}\}$
(3) $x \in \{-\frac{4\pi}{3}, -\frac{2\pi}{3}\}$
(4) $x \in \{\frac{2\pi}{3}, \frac{4\pi}{3}\}$
(5) $x \in \{-3\pi, -\pi, \pi\}$
(6) $x \in \{1.16, 5.12\}$
(7) $x \in \{1.98, 4.30\}$
(8) $x \in \{-5.12, -1.16\}$
(9) $x \in \{-4.30, -1.98\}$
(10) $x \in \{-\pi, \pi\}$

Q3. (1) $x = \pm\frac{2\pi}{3}$ (2) $x \in \{-1.89, 1.25\}$ (3) $x \in \{-2.82, 0.322\}$

Q4. (1) $x \in \{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}\}$
(2) $x \in \{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\}$
(3) $x \in \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, 2.03, 5.18\}$
(4) $x = \frac{\pi}{2}$
(5) $x \in \{60, 180, 300\}$
(6) $x \in \{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\}$
(7) $x \in \{54.7, 125, 235, 305\}$
(8) $x \in \{0, 2\pi\}$
(9) $x \in \{\frac{7\pi}{6}, \frac{11\pi}{6}\}$
(10) $x = \frac{\pi}{2}$
(11) $x \in \{\frac{\pi}{4}, \frac{5\pi}{4}\}$
(12) $x \in \{1.19, \frac{3\pi}{4}, 4.33, \frac{7\pi}{4}\}$
(13) $x \in \{0.841, \frac{2\pi}{3}, \frac{4\pi}{3}, 5.44\}$
(14) $x \in \{\frac{\pi}{4}, 2.55, \frac{5\pi}{4}, 5.70\}$
(15) $x \in \{\frac{\pi}{2}, \frac{3\pi}{2}\}$
(16) $x \in \{0, 120, 240, 360\}$
(17) $x \in \{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}\}$
(18) $x \in \{0, 120, 240, 360\}$
(19) $x \in \{1.16, 1.91, 4.37, 5.12\}$
(20) $x \in \{1.25, 2.68, 4.39, 5.82\}$
(21) $x \in \{\frac{\pi}{3}, \frac{5\pi}{3}, 1.95, 5.09\}$
(22) $x \in \{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}\}$
(23) $x \in \{\frac{\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{3}, \frac{7\pi}{4}\}$

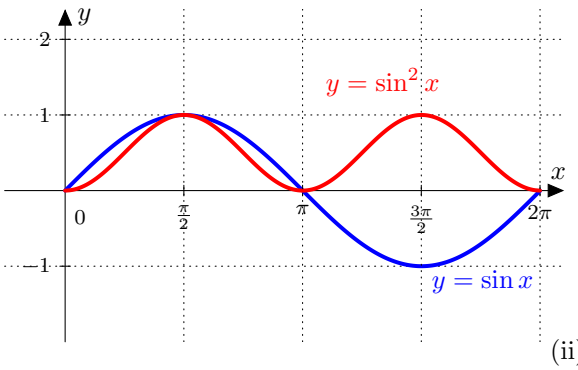
Q5. (1) $x \in \{\frac{\pi}{4}, \frac{7\pi}{4}\}$
(2) $x = \frac{\pi}{3}$
(3) $x = \frac{4\pi}{3}$
(4) $x = \frac{3\pi}{4}$
(5) $x = \frac{k\pi}{18}, k \in \{1, 5, 13, 17, 25, 29\}$
(6) $x = \frac{5\pi}{6}$
(7) $x \in \{0, \frac{\pi}{3}, \pi, \frac{4\pi}{3}, 2\pi\}$
(8) $x \in \{\pi, \frac{5\pi}{3}\}$
(9) $x = \frac{4\pi}{3}$

Q6. (1) $x \in \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\}$
(2) $x = 30k, k \in \{0, 1, 3, 5, 6, 7, 9, 11, 12\}$
(3) $x \in \{0, \pi, 2\pi\}$
(4) $x \in \{\frac{\pi}{4}, \frac{5\pi}{4}\}$
(5) $x = \frac{4\pi}{3}$
(6) $x \in \{0, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi, \frac{3\pi}{2}, 2\pi\}$
(7) $x \in \{0, \pi, 2\pi\}$
(8) $x \in \{\frac{\pi}{2}, \frac{3\pi}{2}\}$
(9) $x \in \{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}\}$
(10) $x \in \{0, \pi, 2\pi\}$
(11) $x \in \{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\}$
(12) $x = k\frac{\pi}{2}, k \in \{0, 1, 2, 3, 4\}$
(13) $x = 120k - 100$ or $120k - 20, k \in \{1, 2, 3\}$ or $x = 60k, k \in \{0, 1, 2, 3, 4, 5, 6\}$
(14) $x \in \{0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}, 2\pi\}$

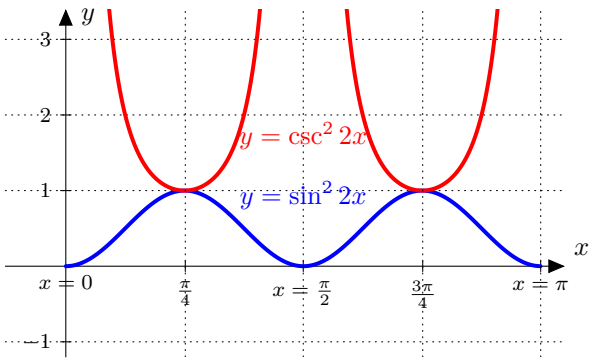
Q7. (1) $x \in]30, 60[\cup]210, 240[$
(2) $x \in [0, 2\pi]$
(3) $\frac{\pi}{6} < x < \frac{5\pi}{6}$
(4) $x \in [0, \frac{\pi}{4}] \cup]\frac{\pi}{2}, \frac{5\pi}{4}] \cup]\frac{3\pi}{2}, 2\pi]$
(5) $x \in \{\frac{\pi}{2}, \frac{3\pi}{2}\}$
(6) $x \in [0, \frac{\pi}{3}] \cup [\frac{2\pi}{3}, \frac{4\pi}{3}] \cup [\frac{5\pi}{3}, 2\pi]$
(7) $x \in [0, 2\pi]$
(8) $\frac{2\pi}{3} < x < \frac{4\pi}{3}$
(9) $x \in [0, 0.927] \cup [5.36, 2\pi]$
(10) $x \in [240, 300]$

Q8. (i) $A = B + 2\pi k, k \in \mathbb{Z}$ or $A = \pi - B + 2\pi k, k \in \mathbb{Z}$
(ii) $x \in \{\frac{\pi}{6}, \frac{\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{11\pi}{6}\}$
(iii) hint: multiply both sides by $\frac{\sqrt{2}}{2}$
$x \in \{0, \frac{\pi}{8}, \frac{5\pi}{8}, \pi, \frac{9\pi}{8}, \frac{13\pi}{8}, 2\pi\}$

Q9. (i)



(ii)



(iii) $\frac{\pi}{4}$.
(v) $(\frac{\pi}{8}, 4), (\frac{3\pi}{8}, 4)$

CHAPTER 17

ARCS, SECTORS, SINE RULE, COSINE RULE

QUESTIONS 17.1.

Q1. 119°

Q2. 60°

Q3. 70.5°

Q4. $A \approx 64.9°, B \approx 47.1°$

Q5. no such triangle

Q6.

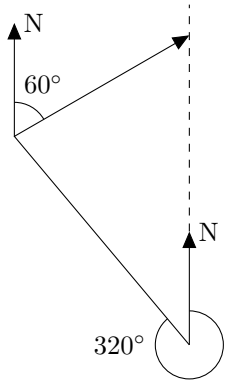
	A	B	C	a	b	c
(1)	48°	42.4°	89.6°	$7.06cm$	$6.4cm$	$9.5cm$
(2)	48°	65°	67°	$7.67cm$	$9.35cm$	$9.5cm$
(3)	45.4°	50.5°	84°	$4.8cm$	$5.2cm$	$6.7cm$
(4)	41.1°	56°	82.9°	$6.5cm$	$8.2cm$	$9.82cm$
(5)	40°	60.4°	79.5°	$3.4cm$	$4.6cm$	$5.2cm$
(6)	51.1°	31°	97.9°	$20.4cm$	$13.5cm$	$26cm$
(7)	54°	47.7°	78.3°	$7.44cm$	$6.8cm$	$9cm$
(8)	54°	61°	65°	$7.94cm$	$8.59cm$	$8.9cm$
(9)	45.4°	50.7°	83.9°	$5.8cm$	$6.3cm$	$8.1cm$
(10)	42.6°	58°	79.4°	$7.5cm$	$9.4cm$	$10.9cm$

Q7. $15.4cm^2$

Q8. $10.6cm^2$

Q9. (a) $1.27cm^2$
(b) $3.67cm$
(c) $9.44cm$

Q10. a) $178km$
b) $273km$



c)

Q11. (i) $4.49km$
(ii) 290°
(iii) 110°

Q12. (i) $5.44km$
(ii) 288°
(iii) 108°

Q13. a) $AB = 4\sqrt{2}cm$
b) $ASB = 0.964(55.2°)$
c) $7.22cm^2$

Q14. $2.63cm^2$

Q15. $B \approx 59.8°$, $C \approx 52.5°$

Q16. $268cm^2$

Q17. 61.7°

Q18. $2.92km$

Q19. (a) 75°
(b) $97.9cm^2$
(c) $7.96cm$
(d) $36.9cm$

Q20. (a) $15\sqrt{2}m$
(b) $5\sqrt{17}m$
(c) 76.0°
(d) 93.4°

17.1 MISCELLANEOUS PROBLEMS

Q21. 81.9°

Q22. 46.9°

Q23. (i) $\sqrt{2}+1$
(ii) $\frac{1}{2}(2+\sqrt{2}+\sqrt{6}2\sqrt{3})$

Q24. —

Q25. —

Q26. $234m$

Q27. 0.9

PART V

DIFFERENTIAL CALCULUS

CHAPTER 18

EVALUATING DERIVATIVES

18.1 DEFINITION OF A DERIVATIVE

QUESTIONS 18.1.

Q1. —

Q2. (1) $\frac{dy}{dx} = 2x$
(2) $\frac{dy}{dx} = 3x^2$
(3) $\frac{dy}{dx} = 4x^3$
(4) $\frac{dy}{dx} = 6x + 2$
(5) $\frac{dy}{dx} = 6x^2$
(6) $\frac{dy}{dx} = -\frac{1}{x^2}$
(7) $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$
(8) $\frac{dy}{dx} = \frac{3}{2}\sqrt{x}$
(9) $\frac{dy}{dx} = \frac{\sqrt{2}}{2\sqrt{x}}$
(10) $\frac{dy}{dx} = -\frac{2}{x^3}$
(11) $\frac{dy}{dx} = 1$
(12) $\frac{dy}{dx} = 0$

Q3. —

18.2 POWER RULE

QUESTIONS 18.2.

Q4. (1) $28x^6 - 12x^3 + 5$
(2) $15x^4 - 6x^2 + 6$
(3) $10x^4 - 15x^2 + 8x + 3$
(4) $-\frac{2}{x^2}$
(5) $\frac{3}{x^3}$
(6) $\frac{4}{x^4}$
(7) $16x^3 - 8x$
(8) $8x^3 - 18x^2 - 2x + 5$
(9) $-9x^{-4} + 6x^3 + 6x^{-5}$
(10) $-\frac{\pi}{2x\sqrt{x}}$
(11) $-\frac{1}{x\sqrt{x}}$
(12) $\frac{3}{x^2} + 3$
(13) $-\frac{1}{2x^2} - \frac{3}{2x^4}$
(14) $-\frac{3\sqrt{2}}{4x^2\sqrt{x}} + \frac{1}{x^2}$
(15) $-\frac{\sqrt{3}}{6x\sqrt{x}} - \frac{2}{x^2}$
(16) $-\frac{5}{4}x^{-\frac{11}{6}} = -\frac{5}{4x\sqrt[6]{x^5}}$
(17) $-\frac{\sqrt{3}}{12}x^{-\frac{7}{6}} - \frac{\sqrt{3x}}{4}$
(18) $-\frac{3\sqrt{2}}{4x\sqrt{x}} + \frac{21}{2x^4\sqrt{x}} - \frac{2}{3x\sqrt[3]{2x}}$

18.3 DERIVATIVES OF ELEMENTARY FUNCTIONS

QUESTIONS 18.3.

Q5. (1) $\frac{dy}{dx} = \sec^2 x$
(2) $\frac{dy}{dx} = -\csc^2 x$
(3) $\frac{\sin x}{\cos^2 x} = \tan x \sec x$
(4) $\frac{\cos x}{\sin^2 x} = -\cot x \csc x$

Q6. (1) $6x\cos x - (3x^2 - 5)\sin x$
(2) $-2\sin x$
(3) $\frac{-4}{(x+1)^3}$
(4) $\frac{\tan x - x\sec^2 x}{\tan^2 x} = \cot x - x\csc^2 x$
(5) $\frac{-2x^2-4}{(x-2)^2(x+1)^2}$
(6) $6x^2t^4 - 2t^3x$
(7) $(10x^4 - 40x^3)\cot x - (2x^5 - 10x^4)\csc^2 x$
(8) $\frac{-2\sin x\tan x - 2\cos x\sec^2 x}{\tan^2 x} = \frac{-2\sin^2 x - 2}{\cos x\tan^2 x}$
(9) $\frac{x^3+3x^2+3x-6}{(x+1)^3} = 1 - \frac{7}{(x+1)^3}$

Q7. (1) $(x^2 + 2x)e^x$
(2) $x^3\cos x + 3x^2\sin x$
(3) $\cos 2x$
(4) $-6x\cos x - (1 - 3x^2)\sin x$
(5) $e^x(\sin x + \cos x)$
(6) $1 + \ln x$
(7) $e^x(\ln x + \frac{1}{x})$
(8) $2\cos 2x$
(9) $-2\sin 2x$
(10) $3\cos 3x$

Q8. $p'(5) = 11$, $q'(5) = -\frac{5}{9}$

18.4 CHAIN RULE

QUESTIONS 18.4.

Q9. (1) $6\sin^2 2x\cos 2x$

(2) $6\cos 3xe^{2\sin 3x}$

(3) $(6x^2-1)(\ln 3)3^{2x^3-x+1}$

(4) $\frac{2\cos x}{\ln 2\sin x}$

(5) $-\frac{(6x^2+1)\sin\ln(2x^3+x-3)}{2x^3+x-3}$

(6) $10(6x+2)(3x^2+2x-1)^9$

(7) $4\sin^3 x\cos^5 x-4\sin^5 x\cos^3 x$

(8) $\frac{\tan^2 x-2x\tan x\sec^2 x}{\tan^4 x}$

(9) $24\pi(2\sin^3\pi x-2)^3\sin^2\pi x\cos\pi x$

(10) $-\frac{2}{5}(1+4x-3x^4)^{-\frac{6}{5}}(4-12x^3)$

(11) $6\sin 3x\cos 3x$

(12) $-2\sin 2xe^{\cos 2x}$

(13) $2x(\ln 3)3^{x^2+1}$

(14) $\frac{2x+2}{(\ln 2)(x^2+2x+3)}$

(15) $\frac{3\cos\ln(2x^3)}{x}$

(16) $30(2x+1)^{14}$

(17) $3\sin^2 x\cos^6 x-5\sin^4 x\cos^4 x$

(18) $36(2\sin^2 3x+1)^2\sin 3x\cos 3x$

(19) $\frac{10}{3}x^4(1-2x^5)^{-\frac{4}{3}}$

(20) $\frac{e^{3x^2+1}(6x-\frac{1}{2\sqrt{x}})}{x}=\frac{e^{3x^2+1}(12x\sqrt{x}-1)}{2x\sqrt{x}}$

(21) $4\left(\frac{2x^2+x-1}{x+2}\right)^3\frac{2x^2+8x+3}{(x+2)^2}$

(22) $\frac{6}{e^{2x}}$

(23) $e^{2x}(2\sin x+\cos x)$

(24) $2\sec^2 x\tan x$

(25) $10x(x+1)(2x-3)^2$

(26) $36x\sin(3x)^2\cos(3x)^2$

(27) $2e^x\cos(2e^x)$

(28) $(\ln\pi)(\pi^{\cos^2 x+1})(-2\cos x\sin x)$

(29) $\frac{(6x-1)(1-\ln(3x^2-x))}{(3x^2-x)^2}$

(30) $\frac{\cos x}{\ln 2\sin x}=\frac{\cot x}{\ln 2}$

(31) 0

(32) $30(\sin 2x)^{14}\cos 2x$

(33) $3\tan^2 x\sec^2 x\cos^5 x-5\tan^3 x\cos^4 x\sin x$
$=3\sin^2 x\cos x-5\sin^4 x\cos x$

(34) $\frac{d}{dx}\frac{x}{\sin x}=\frac{\sin x-x\cos x}{\sin^2 x}$

(35) $3(2e^6x+e^x-1)^2(12e^6x+e^x)$

(36) $\frac{5}{3\sqrt{x}}(1-2\sqrt{x})^{-\frac{8}{3}}$

(37) $2xe^{2x}(x+1)$

(38) $4(\frac{e^x}{\sqrt{x}+1})^3\frac{e^x(\sqrt{x}+1-\frac{1}{2\sqrt{x}})}{(\sqrt{x}+1)^2}=\frac{2e^4x(2x+2\sqrt{x}-1)}{(\sqrt{x}+1)^5\sqrt{x}}$

(39) $\frac{e^{2\ln(4x-2)}}{2x-1}$

(40) $site^2n$

(41) $(12x^3-2)(\ln(3x^4-2x-7)+1)$

(42) $(6x^2+1)(\ln(2x^3+x+1)+1)$

Q10. $h'(1)=-6$, $h'(2)=-2$

Q11. $2\sqrt{3}-\frac{4\pi}{3},-\frac{8\pi}{3}$

Q12. $y=-\frac{1}{24}x+\frac{13}{12}$ or
$y=\frac{1}{24}x+\frac{13}{12}$

18.5 Derivatives of inverse functions

Questions 18.5.

Q13. –

Q14. –

Q15. $\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$

Q16. $\frac{dy}{dx}=-\frac{1}{\sqrt{1-x^2}}$

Q17. (i) $\frac{1}{9e}$

(ii) $\left(\frac{1}{2}e^{-2.5},\frac{1}{2}\right)$

Q18. (i) —

(ii) $-\frac{1}{2}$

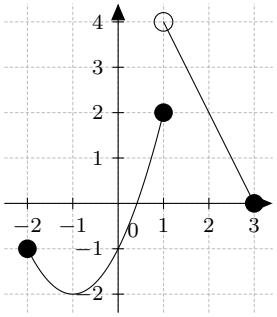
(iii) $(-2\pi,2\pi)$, $(0,0)$, $(2\pi,-2\pi)$

Chapter 19

Piecewise functions

Questions 19.1.

Q1. (i)



(ii) $f(1)=2\neq\lim_{x\to 1^+}f(x)=4$

(iii) $\begin{cases}x^2+2x-1 & \text{for } x\in[-2,1]\\ -2x+4 & \text{for } x\in]1,3]\end{cases}$

(iv) no, it is not continuous at $x=1$, $f'(1)=4\neq\lim_{x\to 1^+}f'(x)=-2$

(v) $y=4x-2$

Q2. (i) $b = 6, c = -3$
(ii) $a = -2, c = -4$
(iii) $a = 2, b = 0$

Q3. (i) —
(ii) $b = 8$, no
(iii) $b = 6, c = -7$

Q4. $b = -12, m = -\frac{3}{2}$

Q5. e.g. $f(x) = \begin{cases} x^2 + 4x + 4 & \text{for } x \in A \\ -x^2 + 2 & \text{for } x \in B \\ x^2 - 4x + 4 & \text{for } x \in C \end{cases}$

CHAPTER 20

LINES TANGENT AND NORMAL TO CURVES

QUESTIONS 20.1.

Q1. (1) $y = 26x - 37$
(2) $y = -\frac{1}{2}x + \frac{1}{2}$
(3) $y = 3ex - 2e$
(4) $y = x$
(5) $y = x$
(6) $y = \frac{3}{2}x + \frac{3}{2} - \frac{\pi}{4}$
(7) $y = -\frac{6}{25}x + \frac{6}{25}\ln 2 + \frac{2}{5}$
(8) $y = 3x - 4$
(9) $y = 2x + 1$

Q2. (1) $y = -\frac{1}{26}x + \frac{196}{13}$
(2) $y = 2x + 3$
(3) $y = -\frac{1}{3e}x + e - \frac{1}{3}e$
(4) $y = -x + \pi$
(5) $y = -x + 2$
(6) $y = -\frac{2}{3}x - \frac{2}{3} + \frac{\pi}{4}$
(7) $y = \frac{25}{6}x - \frac{25}{6}\ln 2 + \frac{2}{5}$
(8) $y = -\frac{1}{3}x + \frac{28}{3}$
(9) $y = -\frac{1}{2}x + 1$

Q3. (a) $y = -4x - 8$ (b) $(-2, 0)$ (c) $y = \frac{1}{4}x - \frac{15}{4}$

Q4. $y = 2x - 3, (2, 1), (3.5, 4)$

Q5. $y = -\frac{1}{2}x + \frac{\pi}{12} + \sqrt{3}$,
$(\frac{\pi}{6}, \sqrt{3}), (1.18, 1.40)$

Q6. Normal at $(0, 0)$: $y = x$
$\frac{dy}{dx} = 0$ at $(\pm 1, \mp\frac{1}{2})$,
$\therefore y = \mp\frac{1}{2}$ respectively
Intersections: $(-\frac{1}{2}, -\frac{1}{2})$ and $(\frac{1}{2}, \frac{1}{2})$

Q7. Normal at $(0, 0)$: $y = 2x$
$\frac{dy}{dx} = 0$ at $(-2, \frac{1}{2}), (2, -\frac{1}{2})$,
$\therefore y = \pm\frac{1}{2}$ respectively
Intersections: $(-\frac{1}{4}, -\frac{1}{2})$ and $(\frac{1}{4}, \frac{1}{2})$.

Q8. $At(2, \sqrt{2}) : y = \frac{\sqrt{2}}{4}x + \frac{\sqrt{2}}{2}$

Q9. $(1, 0), y = 2x - 2$

Q10. $A = B = \pi$

Q11. $A = \frac{1}{3}, y = \frac{1}{3}x + 1$

Q12. $A = 2, B = 4$

Q13. $y = -4x, (-\frac{1}{2}, 2)$

Q14. $c = 4, (1, \frac{1}{2})$

Q15. $(4, -2), y = -0.5x; (3.5, -2.75), y = -x + 0.75$

Q16. $A = \frac{1}{2}, y = -2x + 1$

Q17. $(4, 2), y = -4x + 18$

Q18. $A = \pi, B = -\frac{\pi}{4}$

Q19. $A = 2, (\frac{1}{2}, 0)$

Q20. $y = 4x - 2, (1, 2), (-\frac{9}{8}, -\frac{13}{2})$;
$y = \frac{1}{4}x - \frac{1}{8}, (-\frac{3}{2}, -\frac{1}{2}), (6.78, 1.57)$

Q21. $c = 3, (-1, 1), (4, \frac{8}{3})$

Q22. $a = -4, b = 18$

Q23. $y = ex$

Q24. $y = -\frac{3}{4}x + 1$

CHAPTER 21

FURTHER DERIVATIVES. USE OF DERIVATIVES IN SKETCHING GRAPHS

QUESTIONS 21.1.

Q1. (1) $-\frac{1}{4}e^{-0.5x}(7\cos 2x + 23\sin 2x)$

(2) $(12x+2)\ln(3x+1) + 6x + \frac{3(12x^3+9x^2+2x+9)}{(3x+1)^2}$

(3) $\frac{2(7x^3+6x^2-60x+56)}{(x^2-3x+2)^3}$

Q2. (1) $3^n e^{3x}$

(2) $(-1)^{\lfloor \frac{2n+3}{4} \rfloor} a^n \left((\frac{1+(-1)^{n+1}}{2}) \sin(ax+b) + (\frac{1+(-1)^n}{2}) \cos(ax+b) \right)$

(3) $\frac{1}{a}(-1)^n (n+1)! x^{-n-2}$

Q3. (i) $\frac{dy}{dx} = e^{-\frac{1}{4}x^2+x}(-\frac{1}{2}x+1)$, $\frac{dy}{dx} = 0$ for $x = 2$ only

(ii) $\frac{dy}{dx} > 0$ for $x < 2$

(iii) $\frac{dy}{dx} < 0$ for $x > 2$

(iv) $\frac{d^2y}{dx^2} = \frac{1}{4}e^{-\frac{1}{4}x^2+x}(x^2-4x+2) = -\frac{1}{2}e$ for $x = 2$

(v) $\frac{d^2y}{dx^2} > 0$ for $x < 2-\sqrt{2}$ or $x > 2+\sqrt{2}$,
$\frac{d^2y}{dx^2} < 0$ for $2-\sqrt{2} < x < 2+\sqrt{2}$

Q4.

		(1)	(2)	(3)	(4)
	$\frac{dy}{dx}$	$3x^2-3$	$e^x(x+1)$	$\ln x + 1$	$\frac{2x}{(x^2+1)^2}$
	$\frac{d^2y}{dx^2}$	$6x$	$e^x(x+2)$	$\frac{1}{x}$	$\frac{2-6x^2}{(x^2+1)^3}$
(i)		$0, \pm\sqrt{3}$	0	1	0
(ii)	$y > 0$	$-\sqrt{3} < x < 0, x > \sqrt{3}$	$x > 0$	$x > 1$	$x \neq 0$
	$y < 0$	$x < -\sqrt{3}, 0 < x < \sqrt{3}$	$x < 0$	$0 < x < 1$	none
(iii)		± 1	-1	$\frac{1}{e}$	0
(iv)	$\frac{dy}{dx} > 0$	$x < -1, x > 1$	$x > -1$	$x > \frac{1}{e}$	$x > 0$
	$\frac{dy}{dx} < 0$	$-1 < x < 1$	$x < -1$	$0 < x < \frac{1}{e}$	$x < 0$
(v)		0	-2	none	$\pm\frac{1}{\sqrt{3}}$
(vi)	$\frac{d^2y}{dx^2} > 0$	$x > 0$	$x > -2$	$x > 0$	$-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$
	$\frac{d^2y}{dx^2} < 0$	$x < 0$	$x < -2$	none	$x < -\frac{1}{\sqrt{3}}, x > \frac{1}{\sqrt{3}}$

21.1 Stationary and inflection points

21.2 Kinds of inflection points

Questions 21.2.

Q5. (1) stationary inflection point

(2) minimum

(3) non-stationary inflection point

(4) minimum

(5) maximum

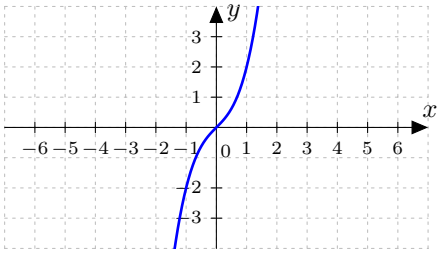
(6) none

(7) non-stationary inflection point

Q6. (i) 0

(ii) $\frac{dy}{dx} = 3x^2 + 1 \neq 0$
$\frac{d^2y}{dx^2} = 6x = 0$ for $x = 0$

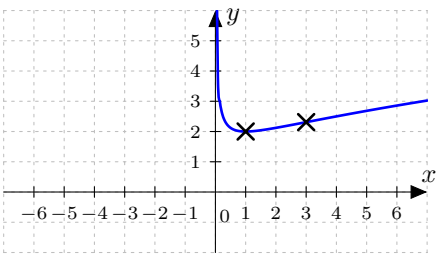
(iii)



Q7. (i) there are no zeroes

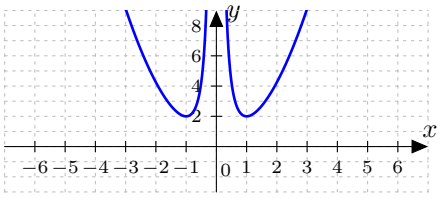
(ii) $\frac{dy}{dx} = \frac{x-1}{2x\sqrt{x}} = 0$ for $x = 1, (1, 2)$
$\frac{d^2y}{dx^2} = \frac{\sqrt{x}(3-x)}{4x^3} = 0$ for $x = 3, (3, \frac{4\sqrt{3}}{3})$

(iii)



Q8. (i) there are no zeroes

(ii) $\frac{dy}{dx} = 2x - \frac{2}{x^3} = 0$
for $x = \pm 1, (1, 2), (-1, 2)$
$\frac{d^2y}{dx^2} = 2 + \frac{6}{x^4} \neq 0$

(iii) 

Q9. $(1 \pm \frac{\sqrt{2}}{2}, \sqrt{e})$

Q10. $(e^{-\frac{1}{3}}, -\frac{1}{3e})$, $\frac{d^2y}{dx^2} > 0$

Q11. *Note that the graphs are not included in the answers to this question*

$\frac{dy}{dx}$	$\frac{d^2y}{dx^2}$	stationary points	inflection points
(1) $e^{-x}(1-x)$	$e^{-x}(x-2)$	$(1, \frac{1}{e})$ - max	$(2, \frac{2}{e^2})$
(2) $e^{-x}(-x^2+2x)$	$e^{-x}(x^2-4x+2)$	$(0,0)$ - min, $(2, \frac{4}{e^2})$ - max	$(2 \pm \sqrt{2}, (2 \pm 2\sqrt{2})e^{-2\mp\sqrt{2}})$
(3) $-e^{-x}(x^2-4x+3)$	$e^{-x}(x^2-6x+7)$	$(1,0)$ - min, $(3, \frac{4}{e^3})$ - max	$(3 \pm \sqrt{2}, (2 \pm 2\sqrt{2})e^{-3\mp\sqrt{2}})$
(4) $\frac{1-\ln 2x}{x^2}$	$\frac{2\ln 2x-3}{x^3}$	$(\frac{1}{2}e, \frac{2}{e})$ - max	$(\frac{1}{2}e^{1.5}, \frac{3}{e^{1.5}})$
(5) $\frac{1-2\ln x}{x^3}$	$\frac{6\ln x-5}{x^4}$	$(\sqrt{e}, \frac{1}{2e})$ - max	$(e^{\frac{5}{6}}, \frac{5}{6e^{\frac{5}{3}}})$
(6) $\frac{e^x(x-1)}{x^2}$	$\frac{e^x(x^3-2x^2+2x)}{x^4}$	$(1, e)$ - min	none
(7) $\frac{e^x(2x-1)}{2x\sqrt{x}}$	$\frac{e^x\sqrt{x}(4x^2-4x+3)}{4x^3}$	$(\frac{1}{2}, \sqrt{2e})$ - min	none
(8) $27x^2-6x-8$	$54x-6$	$(-\frac{4}{9}, 6\frac{14}{81})$ - max, $(\frac{2}{3}, 0)$ - min	$(\frac{1}{9}, 3\frac{7}{81})$
(9) $12(x^3-x^2-4x+4)$	$12(3x^2-2x-4)$	$(-2,-113), (2,15)$- min,$(1,22)$- max	$(\frac{1-\sqrt{13}}{3}, -56.5)$, $(\frac{1+\sqrt{13}}{3}, 18.3)$
(10) $12(x^3-x^2-16x+16)$	$12(3x^2-2x-16)$	$(-4,-1024), (4,0)$- min, $(1,351)$- max	$(-2,-432), (\frac{8}{3}, 161)$
(11) $4x^3-12x-8$	$12x^2-12$	$(2,0)$ - min	$(-1,27), (1,11)$

Q12. (i) $(-1,2), (2-\sqrt{3}, \frac{1+\sqrt{3}}{2}), (2+\sqrt{3}, \frac{1-\sqrt{3}}{2})$ (ii) At $(-1,2): y = -x+1$

CHAPTER 22

APPLICATIONS OF DIFFERENTIAL CALCULUS

22.1 IMPLICIT DIFFERENTIATION

QUESTIONS 22.1.

Q1. (1) $3x^2 - 2y\frac{dy}{dx} = 4xy + 2x^2\frac{dy}{dx}$

(2) $2x + 4y + 4x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$

(3) $\sec^2(x+y)(1+\frac{dy}{dx}) - \sec^2(xy^2)(y^2 + 2xy\frac{dy}{dx}) = 0$

(4) $\frac{2y\frac{dy}{dx}}{1+y^4} = 1 + y + x\frac{dy}{dx}$

(5) $\frac{y+x\frac{dy}{dx}}{\sqrt{xy}(1+4xy)} = \frac{y-\frac{dy}{dx}x}{y^2}$

(6) $\frac{y+x\frac{dy}{dx}}{1+x^2y^2} = 1 + \frac{dy}{dx}$

(7) $\cos(x^2y)(2xy + x^2\frac{dy}{dx}) - 2\cos(xy)\sin(xy)(y + x\frac{dy}{dx}) = 0$

(8) $\frac{y(2xy-x^2\frac{dy}{dx})}{x^2y^2} = 1$

(9) $3e^{xy}(y + x\frac{dy}{dx}) = 0$

(10) $e^{x\cos y}(\cos y - x\sin y\frac{dy}{dx}) = 0$

(11) $\frac{2xy+x^2\frac{dy}{dx}}{x^2y} = 0$

(12) $\cos(x^3y)(3x^2y + x^3\frac{dy}{dx}) - \sin(xy^3)(y^3 + 3xy^2\frac{dy}{dx}) = 0$

(13) $e^{2xy}(2y + 2x\frac{dy}{dx}) = \frac{e^x(e^y - e^y\frac{dy}{dx}+1)}{(e^y+1)^2}$

Q2. $\frac{43}{34}$

Q3. @$(1,-1)$, $\frac{dy}{dx} = \frac{1}{2}$; @$(1,1)$, $\frac{dy}{dx} = -\frac{1}{2}$

Q4. $-\frac{19}{4}$

Q5. (1) $\frac{dy}{dx} = \frac{3x^4y - x^2y^2 - y^2}{x^5}$

(2) $\frac{dy}{dx} = \frac{ye^{xy}}{1 - xe^{xy}}$

Q6. (1) $\frac{dy}{dx} = \frac{y}{1-x}; \frac{d^2y}{dx^2} = \frac{2y}{(1-x)^2}$

(2) $\frac{dy}{dx} = -\frac{y+\sqrt{y}}{x}; \frac{d^2y}{dx^2} = \frac{4y+5\sqrt{y}+1}{2x^2}$

Q7. —

Q8. (i) —

(ii) -3

(iii) 2

(iv) decreasing and concave up

Q9. (i) $-3 \leq x \leq 3, -2 \leq y \leq 2$

(ii) $\frac{dy}{dx} = -\frac{4x}{9y}$

(iii) $\frac{dy}{dx} = -\frac{4x}{9y} = \frac{-4x}{\pm 3\sqrt{36-4x^2}}$

(iv) An ellipse

Q10. (i) $x \geq -1, y \in \mathbb{R}$

(ii) $\frac{dy}{dx} = \frac{1}{2y}$

(iii) $\frac{dy}{dx} = \frac{1}{\pm 2\sqrt{x+1}}$

(iv) parabola symmetrical in x-axis

Q11. $e^{xy^2} = 3x^2y - 6xy^2$
$\therefore e^{xy^2}(y^2 + 2xy\frac{dy}{dx}) =$
$= 6xy + 3x^2\frac{dy}{dx} - 6y^2 - 12xy\frac{dy}{dx}$

Q12. (1) $\pm\frac{4}{5}$

(2) $-\frac{37}{26}$

22.2 Curves defined parametrically

Questions 22.2.

Q13. (i) $-5 \leq x, y \leq 5$

(ii) $\frac{dy}{dt} = 5\cos t, \frac{dx}{dt} = -5\sin t, \frac{dy}{dx} = -\cot t$

(iii) —

(iv) —

(v) $(\frac{5\sqrt{2}}{2}, \pm\frac{5\sqrt{2}}{2}), (-\frac{5\sqrt{2}}{2}, \pm\frac{5\sqrt{2}}{2})$

(vi) circle with centre at origin and radius 5

(vii) $x^2 + y^2 = 25$

Q14. (i) $\frac{dy}{dx} = -\cot t$

(ii) circle radius 1, centre at $(2, -3); (x-2)^2 + (y+3)^2 = 1$

Q15. (i) $\frac{\pi}{3}, \frac{5\pi}{3}$

(ii) $\frac{dy}{dx} = \frac{1}{8\sin^3 t}; P_1(1, \frac{2}{\sqrt{3}}), y = \frac{1}{3\sqrt{3}}x + \frac{5}{3\sqrt{3}}; P_2(1, -\frac{2}{\sqrt{3}}, y = -\frac{1}{3\sqrt{3}}x - \frac{5}{3\sqrt{3}}$

(iii) $x^2 = \frac{4\sqrt{y^2-1}}{y^3}$

Q16. (i) $t = \frac{\pi}{6}, x = \frac{3\sqrt{3}}{2}$ or $t = \frac{5\pi}{6}, x = -\frac{3\sqrt{3}}{2}$

(ii) $\frac{dy}{dx} = -4/3\cot t, y = -\frac{4\sqrt{3}}{3}x + 5$ or $y = \frac{4\sqrt{3}}{3}x + 5$

(iii) Ellipse 6×8, centre $(0, -3)$, $\frac{x^2}{9} + \frac{(y+3)^2}{16} = 1$

Q17. (i) $\pm 2\sqrt{6}$

(ii) $\frac{dy}{dx} = -\frac{\sqrt{2}}{2}\cot t$, normal is $y = \sqrt{6}x$ or $y = -\sqrt{6}x$, so it passes through $(0, 0)$

(iii) Ellipse $16 \times 8\sqrt{2}$, centre $(-2, 0)$, $\frac{(x+2)^2}{64} + \frac{y^2}{32} = 1$

Q18. (i) $-4 \pm \sqrt{2}$

(ii) $\frac{dy}{dx} = \csc t, (-4, 2)$

(iii) Hyperbola; asymptotes $y = \pm(x+4)$; zeroes $-5, -3. y^2 = (x+4)^2 - 1$

Q19. $y = -2x - 6, y = 2x + 8$

Q20. —

Q21. (i) —

(ii) $y = 8x - 7.5$

(iii) $y = \frac{x+1}{3x+1}$

Q22. At $(\pm 4\sqrt{2}, 6)$ the tangent is $y = 6$, at $(\pm 4\sqrt{2}, -6)$ the tangent is $y = -6$.

22.3 Optimisation

Questions 22.3.

Q23. 15×15 **Q25.** $5 \times 20 \times 20$ **Q27.** $(4, 1)$ **Q29.** $(\frac{1}{\sqrt{3}}, -\frac{2}{3})$

Q24. 15×30 **Q26.** $\frac{256}{27}$ **Q28.** $20cm \times 10cm$

22.4 Applying derivatives in kinematics

Questions 22.4.

Q30. (i) 4 seconds
(ii) $-1km$
(iii) $1km$
(iv) $2km$
(v) $0.25\frac{km}{s}$
(vi) $0.25\frac{km}{s}$
(vii) $a = 0$

Q31. (i) $1m$
(ii) $15m$
(iii) $14m$
(iv) (a) $3.5\frac{m}{s}$
(b) $4\frac{m}{s}$
(c) $3.5\frac{m}{s}$
(d) $3\frac{m}{s}$
(e) $2.75\frac{m}{s}$
(f) $2.55\frac{m}{s}$
(g) $2.525\frac{m}{s}$
(h) $2.505\frac{m}{s}$
(v) $2.5\frac{m}{s}$
(vi) constantly $= 1$

Q32. (i) $v = e^{-\frac{t}{2}}(4 - 2t)$
(ii) $t = 2s$
(iii) $8/e \approx 2.94m$
(iv) $\frac{32}{e^4} \approx 0.586m$
(v) $a = e^{-\frac{t}{2}}(t - 4)$

Q33. —

Q34. (i) $1.42 \leq t \leq 2.58$
(ii) $-1ms^{-1}$
(iii) $\frac{1}{4}t^4 - 2t^3 + \frac{11}{2}t^2 - 13.5$

Q35. (i) 4
(ii) (a) 5
(b) 9
(iii) 5
(iv) —
(v) —

Q36. (i) $6m$
(ii) $20m$
(iii) $100\pi mh^{-1}$
(iv) $200mh^{-1}$

22.5 Related rates

Questions 22.5.

Q37. $\frac{3}{121} \approx 0.0248(cms^{-1})$

Q38. $\frac{dz}{dt} = -36.4kmh^{-1}$

Q39. $4\pi \approx 12.6(ms^{-1})$ / $\frac{25}{4}\pi \approx 19.6(ms^{-1})$

Q40. $\frac{dp}{dt} = 100$

Q41. (a) -51.8
(b) -219

Part VI

Integral calculus

Task

(1) $y = x^3 + 2x^2 - 2x + C$

(2) $y = 2e^{2x} + \frac{3}{2}x^2 + C$

(3) $y = \frac{2}{3}x\sqrt{x} + C$

(4) $y = -\frac{3}{x} - \frac{1}{x^2} + C$

(5) $y = 3\ln x + \frac{1}{3}\ln x + C$

Chapter 23

Integrals

23.1 Indefinite integrals

Questions 23.1.

Q1. (all answers should include "$+C$")

(1) x^2
(2) $\frac{x^5}{5}$
(3) $\frac{1}{2}x^4$
(4) $\frac{1}{2}x^6$
(5) $\frac{6}{11}x^{5.5}$
(6) $\frac{6}{7}x^{\frac{7}{3}}$
(7) $\frac{5}{2}x^4 - \frac{5}{3}x^3$
(8) $-\frac{3}{2x^2}$
(9) $-\frac{1}{2x^4}$
(10) $\frac{4}{5}x^2\sqrt{x}$
(11) $-\frac{4}{\sqrt{x}}$
(12) $\frac{2\sqrt{2}}{11}x^5\sqrt{x}$
(13) $\frac{6}{5}x^2\sqrt{x}$
(14) $\frac{1}{2}x^2 + 3x$
(15) $\frac{1}{4}x^4 + \frac{3}{2}x^2 + 3\ln x - \frac{1}{2}x^{-2}$
(16) $\frac{1}{2}x^2 + 6x + 12\ln x - \frac{8}{x}$
(17) $\frac{4}{3}x\sqrt{x} - \frac{3}{2}\ln x$
(18) $\frac{3\times 2^{\frac{11}{6}}}{13}x^{\frac{25}{6}}$
(19) $\frac{1}{8}x^4 + \frac{3}{2}x - \frac{9}{16}x^{-2}$
(20) $-\frac{2}{\sqrt{t}} - 6\sqrt{t}$
(21) $\frac{y^3}{3} - 4y - \frac{4}{y}$
(22) $\frac{2}{5}u^2\sqrt{u} - \frac{4}{3}u\sqrt{u} + 2\sqrt{u}$
(23) $\frac{6}{7}\sqrt[3]{v^7} - 12\sqrt[4]{v}$
(24) $\frac{1}{4}a^4 - \ln a$
(25) $\frac{4}{9}x^4\sqrt{x} - \frac{4}{13}x^6\sqrt{x}$
(26) $\frac{3}{2}x^4 - 2x^3 + \frac{3}{2}x^2 + 4x$
(27) $x^2 - \frac{4}{3}x\sqrt{x} + \frac{1}{2}x$
(28) $-\frac{2}{\sqrt{x}} - \frac{2\sqrt{2}}{3}x\sqrt{x}$
(29) $2\sqrt{3x} - \frac{2}{3\sqrt{3}}x\sqrt{x}$

More basic integrals

Questions 23.2.

Q2. (all answers should include "$+C$")

(1) $\frac{1}{2}\ln x$ or $\frac{1}{2}\ln 2x$
(2) $-\frac{1}{5}\cos 5x$
(3) $\frac{1}{21}(3x-2)^7$
(4) $\frac{1}{12}(2x+5)^6$
(5) $\frac{1}{3}(2x+3)^{\frac{3}{2}}$
(6) $\frac{1}{3}e^{3x}$
(7) $\frac{1}{3}e^{3x+1}$
(8) $-\frac{1}{3}e^{1-3x}$
(9) $\frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x}$
(10) $\frac{1}{\pi}\sin(\pi x)$
(11) $x - \frac{1}{2}\cos 2x$
(12) $\frac{1}{\ln 2}2^x$
(13) $\frac{1}{2\ln 2}2^{2x}$
(14) $\frac{1}{2\ln 2}2^{2x+1}$
(15) $\frac{1}{2}\tan 2x$
(16) $-\frac{1}{2}\cos(2x-\pi)$
(17) $2\sin(\frac{x-\pi}{2})$
(18) $e^x + 4x - 4e^{-x}$
(19) $\frac{2}{\pi}\sin(\frac{\pi x-\pi}{2})$
(20) $\cos(\pi - x)$
(21) $\ln(x+2)$
(22) $\frac{1}{2}\ln(2x+4)$
(23) $-\ln(3-2x)$
(24) $-\frac{1}{2}\tan(\pi - 2x)$
(25) $\frac{1}{2}e^{2x} + \frac{1}{2}x^2$
(26) $-\frac{1}{2}\cos 2x + \frac{1}{3}\tan 3x$
(27) $\frac{1}{2}\ln|2x+1|$
(28) $\frac{1}{2\ln 3}3^{2x}$
(29) $\frac{\sqrt{100x-1}}{75}(50x-2)$
(30) $\frac{1}{4}\sin(4x+2) - e^{-x}$
(31) $\frac{1}{2}(\ln 2)(\ln(2x-1))$
(32) $\frac{1}{3\ln 5}5^{3x+4}$

Integration with use of trigonometric identities

Questions 23.3.

Q3. (1) $\frac{9}{2}x + 4\sin x + \frac{1}{4}\sin 2x$ (2) $\frac{x}{2} - \frac{1}{8}\sin 4x$ (3) $\frac{5}{2}x - \frac{3}{4}\sin 2x + \cos 2x$

Q4. (1) $\frac{1}{10}(5\cos x - \cos 5x) + c$ (2) $-\frac{1}{10}\cos 5x - \frac{1}{6}\cos 3x + c$ (3) $-\frac{1}{10}(5\cos x + \cos 5x) + c$

Q5. (1) $\frac{1}{10}(5\sin x + \sin 5x) + c$
(2) $\frac{1}{30}(5\sin 3x + 3\sin 5x) + c$
(3) $\frac{1}{42}(7\sin 3x + 3\sin 7x) + c$
(4) $\frac{1}{10}(5\sin x - \sin 5x) + c$
(5) $\frac{1}{30}(5\sin 3x - 3\sin 5x) + c$
(6) $\frac{1}{42}(7\sin 3x - 3\sin 7x) + c$

Integrals of inverse trigonometric functions

Questions 23.4.

Q6. (all answers should include "$+C$")

(1) $\frac{1}{2}\arctan 2x$
(2) $\frac{1}{\sqrt{2}}\arctan\sqrt{2}x$
(3) $2\arctan(x/2)$
(4) $\frac{3}{\sqrt{2}}\arctan\frac{x}{\sqrt{2}}$
(5) $2\arctan(x-1)$
(6) $\arctan(x+1)$
(7) $\frac{1}{2}\arctan(2x+3)$
(8) $\frac{1}{\sqrt{2}}\arcsin(\sqrt{2}x)$
(9) $\frac{1}{2}\arcsin(\sqrt{2}x)$
(10) $\arcsin(x+1)$
(11) $\arcsin(x-1)$
(12) $\frac{1}{2}\arcsin(2x-1)$

Q7. (all answers should include "$+C$")

(1) $\frac{1}{3}x^3 + 4x\sqrt{x} + 9\ln x$
(2) $\frac{1}{3}(x-2)^{\frac{3}{2}}$
(3) $2\sqrt{2(x-2)}$
(4) $-\frac{1}{3}\cos 3x$
(5) $2\tan 0.5x$
(6) $\frac{1}{12}\sin 6x + \frac{1}{2}x$
(7) $4\arctan x$
(8) $\frac{1}{2\sqrt{2}}\arctan\sqrt{2}x$
(9) $\frac{1}{3\ln 2}2^{3x}$
(10) $\frac{2}{\ln 2}(\sqrt{2})^x$
(11) $\frac{2}{3}\ln(3x-6)$
(12) $\frac{1}{3}e^{3x+2} - 2\sqrt{e^x}$

23.2 Definite integrals

Questions 23.5.

Q1. $y = x^2 + \cos x + 1$

Q2. $y = e^x - x^2 + 2$

Q3. $\frac{2}{3}\sin(3x - \frac{\pi}{2}) + \frac{3x}{\pi}$

Q4. $y = -2\cos(2x - \frac{\pi}{2}) + \frac{16}{\pi}x - 2 + \sqrt{2}$

Q5. $y = -2\sin(\pi x) - 1$

Q6. (1) $\sin x + x\cos x$ (2) $x\sin x + \cos x + C$ (3) -2

Q7. (a) $e^x + xe^x$ (b) $xe^x - e^x + c$ (c) e^2

Q8. (1) 4 (2) 4

Questions 23.6.

Q9. (1) 4
(2) 99
(3) $-\frac{52}{3}$
(4) 18
(5) $\frac{112}{3}$
(6) 96.8
(7) 1
(8) 1
(9) $\frac{3}{8}$
(10) 2
(11) $\frac{7}{6}$
(12) $3 + \ln 2$
(13) $\frac{7}{6}$
(14) $\frac{3}{2} + 2\ln 2$
(15) $\frac{20}{3}$

Questions 23.7.

Q10. (1) 18
(2) -12
(3) -18
(4) 9
(5) -10
(6) $\frac{106}{3}$

Q11. (1) 32
(2) $\frac{14}{3}$
(3) 1
(4) $\frac{1}{2}$
(5) Does not exist
(6) $\frac{1}{2}$
(7) $\frac{1}{2} + \frac{\sqrt{3}}{2}$
(8) $\frac{3}{2}$
(9) $\frac{\pi}{4}$
(10) $\frac{\pi}{8}$
(11) $\frac{\pi}{36}$
(12) $\frac{1}{e}\ln(\frac{3}{2})$
(13) $16 - 2\ln 3$
(14) 63

Q12. $\frac{e^2+3}{2(e^2-1)}$

Q13. (1) $\frac{1}{2}\pi\ln 3$ (2) $155\frac{1}{4}$

Chapter 24

Area and volume

24.1 Area under a curve

Questions 24.1.

Q1. —

Q2. (1) $\frac{64}{3}$ (2) 8 (3) $\frac{937}{12}$

Q3. $\frac{1}{2}(e^2 - 1) \approx 3.19$

Q4. $\frac{7}{3}$

24.2 Area between curves

Questions 24.2.

Q5. $\frac{125}{6}$ **Q6.** $\frac{16}{3}$ **Q7.** $4\sqrt{3}$ **Q8.** 0.201

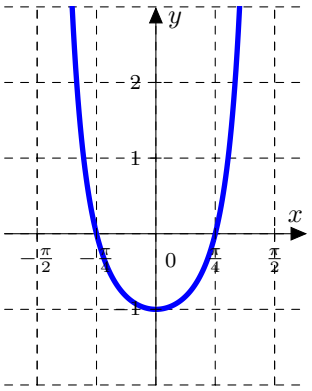
Q9. (i) $f'(x) = -x + 2$
(ii) 2
(iii) $-\frac{1}{2}$
(iv) $y = -\frac{1}{2}x + 1.5$
(v) $(5, -1)$
(vi) $\int_0^5 f(x) + \frac{1}{2}x - 1.5dx =$
$= \int_0^5 (-0.5x^2 + 2.5x)dx$
(vii) $\frac{125}{12} \approx 10.4$

Q10. 1.60

Q11. $\frac{\pi}{2} - \frac{1}{3}$

Q12. (i) $y = \cos 2x$ - lower, $y = \cos^2 x$ - upper
(ii) $A(0,1), B(\frac{\pi}{4}, 0), C(\frac{\pi}{2}, 0)$,
$D(\frac{\pi}{2}, -1), E(\frac{3\pi}{4}, 0), F(\pi, 1)$
(iii) $\int_0^{\pi} (\cos^2 x - \cos 2x)dx$
(iv) $\frac{\pi}{2}$

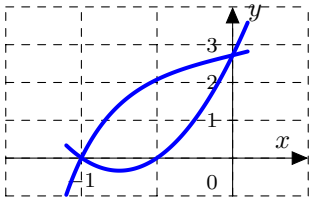
Q13. (i) $k = \frac{\pi}{2}$
(ii)



(iii) $y = \frac{1}{4}x + \frac{\pi}{16}$
(iv) $(0.872, 0.414)$
(v) $\int_{-\frac{\pi}{4}}^{0.872} (\frac{1}{4}x + \frac{\pi}{16} - \sec^2 x + 2)dx$
(vi) 1.4680

Q14. 0.0917

Q15. (i) –



(ii) $g'(x) = e^{x^2}(2x^2 + 1)$;
g is increasing as $g'(x) > 0$
for all x (both factors +ive)
(iii) $g''(x) = e^{x^2}(4x^3 + 6x)$, $Q(0, e)$
(iv)
(v) $\frac{d}{dx}e^{x^2} = 2xe^{x^2}$;
$\int (xe^{x^2} + e)dx =$
$= \frac{1}{2}e^{x^2} + ex + C$
(vi) Area$= \frac{1}{2} + \frac{1}{3}e$

Q16. (i) $y = x - 2$
(ii) $(-2.73, -4.73), (0.732, -1.27)$
(iii) $\int_{-2.73}^{0} (x^3 + 2x^2 - x - 2) - (x - 2)dx + \int_0^{0.732} (x - 2) - (x^3 + 2x^2 - x - 2)dx$
(iv) 7.33

Q17. —

Q18. (a) 2 : 1
(b) n : 1

24.3 Volume of revolution

Questions 24.3.

Q19. $\frac{52}{3}\pi$

Q20. $\frac{16}{15}\pi \approx 3.35$

Q21. π

Q22. (1) $\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$
(2) $\int \sin^2 x dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$
(3) $\frac{\pi^2}{2}$

Q23. (1) $-\frac{1}{2}x + \frac{1}{2}\ln 2$
(2) 0.115
(3) 0.0537

Q24. (1) 1
(2) $\frac{\pi^2}{4}$

Questions 24.4.

Q25. (1) $\frac{43}{6}$
(2) $\frac{101}{3}\pi$
(3) $\frac{706}{15}\pi$

Q26. (1) tangent: $y = 2x - 1.5$, area $= \frac{7}{48}$
(2) $\frac{29}{240}\pi \approx 0.380$
(3) 0.949

Q27. (1) $2\ln 2 - 1 \approx 0.386$
(2) $\frac{\pi}{2} \approx 1.57$
(3) 0.592

Q28. 1.47 or $\frac{\pi}{4}(\pi^2 - 8)$

24.4 Kinematics

Questions 24.5.

Q29. (i) after $4.47h$ (ii) $89.4kmh^{-1}$ (iii) after $6.32h$

Q30. (i) after $2.53h$ (ii) $98.0kmh^{-1}$ (iii) after $4.33h$

Q31. (i) after $2.95h$ (ii) $88.3kmh^{-1}$ (iii) after $4.87h$

Q32. (i) after $3.37h$ (ii) $61.0kmh^{-1}$ (iii) after $13.2h$, on the way back

Q33. (i) $-0.04\pi ms^{-2}$ (ii) $\frac{800}{\pi}m \approx 255m$ (iii) $\frac{2400}{\pi}m \approx 764m$

Q34. (i) at $t = 2h$
(ii) at $t = 6h$
(iii) $v = 12kmh^{-1}$ (at $t = 2h$)
(iv) $v = 36kmh^{-1}$ $(-36kmh^{-1})$ (at $t = 10h$)
(v) $64km$
(vi) $0km$
(vii) $118km$

Chapter 25

Further integration

25.1 Integration by parts

(all answers should include "$+C$")

Introducing integration by parts

Questions 25.1.

Q1. (1) $x\sin x + \cos x$
(2) $-x\cos x + \sin x$
(3) $e^x(x-1)$
(4) $\frac{1}{4}(\sin 2x - 2x\cos 2x)$
(5) $\frac{3}{4}e^{2x-1}(2x-1)$
(6) $\frac{1}{4}((2x^2-1)\sin 2x + 2x\cos 2x)$
(7) $-e^{-x}(x+3)$
(8) $(-x^2+x+2)\cos x + (2x-1)\sin x$
(9) $(3x^2-8)\sin x - (x^3-8x+5)\cos x$
(10) $\frac{1}{4}((2x^2-2x+5)\sin 2x + (2x-1)\cos 2x)$

Exceptions to standard approach

Questions 25.2.

Q2. (1) $x \ln x - x$
(2) $2\sqrt{x}(\ln x - 2)$
(3) $\frac{1}{2}((x^2+1)\arctan x - x)$
(4) $\frac{1}{16}x^4(4\ln 2x - 1)$
(5) $x\ln(x+x^2) - 2x + \ln(x+1)$
(6) $\sin x(\ln \sin x - 1)$

Q3. (1) $\frac{1}{4}x^2(2\ln x - 1)$
(2) $\frac{1}{9}x^3(3\ln x - 1)$

Repeated integration by parts

Questions 25.3.

Q4. (1) $\frac{1}{2}e^x(\sin x + \cos x)$
(2) $\frac{1}{5}e^{2x}(2\sin x - \cos x)$
(3) $\frac{1}{10}e^(-x)(3\sin 3x - \cos 3x)$
(4) $\frac{1}{3}(\cos x \cos 2x + 2\sin x \sin 2x)$
(5) $\frac{1}{13}e^{3x}(3\sin 2x - 2\cos 2x)$
(6) $\frac{x}{2}(\sin(\ln x) - \cos(\ln x)$
(7) $\cos x(1 - \ln(\cos x))$
(8) $\frac{1}{4}\sin 2x(2\ln(\sin x) - 1) - \frac{1}{2}x$

Q5. (1) $\frac{7\pi}{24} - \frac{\sqrt{3}}{8}$
(2) $\frac{\pi}{6} - \frac{\pi^2}{24\sqrt{3}} - 2 + \sqrt{3}$
(3) $2\ln 2 - \frac{3}{4}$
(4) $\frac{8}{15}$

25.2 Integration by substitution

Questions 25.4.

(all answers should include "$+C$")

Q6. (1) $\ln(x^3+2)$
(2) $\frac{1}{4}(x^2+1)^6$
(3) $\frac{1}{3}(\ln(2x-3))^{\frac{3}{2}}$
(4) $\frac{2}{3}(\ln(x^2-3x+2))^{\frac{3}{2}}$
(5) $\frac{1}{3}(2x+1)^{\frac{3}{2}}$
(6) $\frac{1}{15}(2x+1)^{\frac{3}{2}}(3x-1)$
(7) $-\frac{4}{15}(1-x)^{\frac{3}{2}}(3x+2)$
(8) $\frac{4}{3}(3+\sin\frac{1}{2}x)^{\frac{3}{2}}$
(9) $\frac{1}{9}\tan^3 3x$
(10) e^{x^2+1}
(11) $-\frac{1}{3}\cos 3x^2$
(12) $-\cos\ln(x+1)$
(13) $\frac{1}{3}\ln(2-\cos 3x)$
(14) $\frac{2}{3}\sqrt{x^3+2}$

Q7. (1) $\frac{1}{4}\sin^4 x$
(2) $-\ln\cos x = \ln\sec x$
(3) $\frac{1}{2}\tan^2 x$
(4) $\frac{1}{4}\tan^4 x$
(5) $-\frac{1}{5}\cos^5 x$
(6) $-\cos x + \frac{1}{3}\cos^3 x$
(7) $\sin x - \frac{1}{3}\sin^3 x$
(8) $\sin x + \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x$
(9) $\frac{1}{6}\tan^6 x + \frac{1}{4}\tan^4 x$
(10) $\frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x$
(11) e^{e^x}
(12) $\frac{1}{3}\ln^3 x$
(13) $\frac{1}{3}\sin^3 x$
(14) $\cos\cos x$
(15) $e^{\sqrt{x}}$
(16) $-\sin\frac{1}{x}$
(17) $-2\cos\sqrt{x}$
(18) $\sqrt{x}\ln x - 2\sqrt{x}$

Q8. (1) $\tan x - x$
(2) $\ln|\sec x|$
(3) $\frac{1}{2}\tan^2 x + \ln\cos x$
(4) $\frac{1}{2}\tan^2\ln x + \ln\cos\ln x$
(5) $\frac{1}{3}\tan^3 x$
(6) $\frac{1}{3}\tan^3 x - 2\tan x + 2x$
(7) $-\frac{1}{2}\cot 2x - x$

Definite integration by substitution

Questions 25.5.

Q9. (1) 2
(2) $-\frac{1}{2}\ln 2$
(3) $-\frac{1}{2}\ln 2$
(4) $\frac{1}{2}$
(5) 3333.3
(6) $-\frac{1}{\pi}$
(7) $\frac{e-1}{3e}$
(8) $-\frac{1}{3}$
(9) $\frac{\pi^2}{9}$
(10) $\frac{3\pi}{8}$
(11) $\frac{1}{32}(8\pi - \pi^2 - 16\ln 2)$
(12) $\frac{1}{4}(\pi - \ln 4)$
(13) $\frac{1}{4}\ln 2 - \frac{\pi\sqrt{3}}{12}$
(14) $\frac{1}{108}(8\sqrt{3}-9)\pi^{1.5}$

Trigonometric substitutions

Questions 25.6.

Q10. – **Q11.** $\frac{\pi}{4}$ **Q12.** 2π **Q13.** $-\frac{1}{8\sqrt{2}}(2+\pi)$ **Q14.** $\frac{\pi}{3}$

Q15. (1) $\arcsin x + C$
(2) $-\sqrt{1-x^2} + C$
(3) $\frac{1}{2}(\arcsin x - x\sqrt{1-x^2}) + C$
(4) $-\frac{1}{3}\sqrt{1-x^2}(x^2+2) + C$

Q16. 2π

Q17. (1) $\frac{1}{2}x\sqrt{x^2-9} - \frac{9}{2}\ln\left(\sqrt{x^2-9}+x\right) + c$
(2) $\frac{1}{2}x\sqrt{9-x^2} + \frac{9}{2}\arcsin\left(\frac{x}{3}\right) + c$
(3) $\frac{1}{2}x\sqrt{x^2+9} + \frac{9}{2}\ln\left(\sqrt{x^2+9}+x\right) + c$
(4) $\ln(x+\sqrt{x^2+4}) + c$
(5) $\sqrt{x^2+4} + c$
(6) $\frac{1}{2}x\sqrt{x^2+4} - 2\ln\left(\sqrt{x^2+4}+x\right) + c$
(7) $-\frac{1}{\sqrt{x^2+4}} + c$
(8) $\frac{x}{4\sqrt{x^2+4}} + c$
(9) $\frac{1}{\sqrt{4-x^2}} + c$

Q18. (1) $\frac{1}{2}(x\sqrt{x^2-1} - \ln(x+\sqrt{x^2-1})) + C$
(2) $\frac{1}{2}(x\sqrt{x^2+1} + \ln(x+\sqrt{x^2+1})) + C$
(3) $\frac{1}{2}x\sqrt{4-x^2} + 2\arcsin(\frac{x}{2}) + C$
(4) $\frac{1}{2}x\sqrt{x^2-4} - 2\ln(x+\sqrt{x^2-4}) + C$
(5) $\frac{1}{2}x\sqrt{x^2+4} + 2\ln(x+\sqrt{x^2+4}) + C$
(6) $\frac{1}{2}x\sqrt{9-4x^2} + \frac{9}{4}\arcsin(\frac{2x}{3}) + C$
(7) $\frac{1}{2}x\sqrt{4x^2-9} - \frac{9}{4}\ln(2x+\sqrt{4x^2-9})) + C$
(8) $\frac{1}{2}x\sqrt{4x^2+9} + \frac{9}{4}\ln(2x+\sqrt{4x^2+9}) + C$
(9) $\arcsin x + C$
(10) $\ln(x+\sqrt{x^2+1}) + C$
(11) $\ln(x+\sqrt{x^2-1}) + C$
(12) $\arcsin(\frac{x}{2}) + C$
(13) $\ln(x+\sqrt{x^2+4}) + C$
(14) $\ln(x+\sqrt{x^2-4}) + C$
(15) $\frac{1}{2}\arcsin(2x) + C$
(16) $\frac{1}{2}\ln(2x+\sqrt{4x^2+1}) + C$
(17) $\frac{1}{2}\ln(2x+\sqrt{4x^2-1}) + C$

Q19. (1) $\sqrt{1-x^2} + x\arcsin x$
(2) $\frac{1}{4}(x\sqrt{1-x^2} + (2x^2-1)\arcsin x)$
(3) $x\arctan x - \frac{1}{2}\ln(x^2+1)$
(4) $x\arccos x - \sqrt{1-x^2}$

25.3 Integrating rational expressions

Questions 25.7.

Q20. (1) $\frac{1}{4}\arctan(\frac{x}{2})$
(2) $\frac{2}{\sqrt{3}}\arctan(\frac{x}{\sqrt{3}}$
(3) $\arctan(x+3)$
(4) $\frac{1}{4}\arctan(x-0.5)$
(5) $\arctan(x+3)$
(6) $\frac{1}{2}\arctan(\frac{x-4}{2})$
(7) $\frac{2}{\sqrt{3}}\arctan(\frac{2x+3}{\sqrt{3}})$
(8) $-\frac{1}{\sqrt{2}}\arctan(\frac{x-2}{\sqrt{2}})$
(9) $\frac{1}{\sqrt{6}}\arctan(\frac{x\sqrt{3}+2\sqrt{3}}{\sqrt{2}})$
(10) $\frac{1}{3}\arctan(\frac{2x-5}{3})$
(11) $-\frac{1}{3(3x+2)}$
(12) $\frac{1}{2-x}$
(13) $-\frac{1}{x+5}$
(14) $-\frac{1}{3(3x-2)}$
(15) $-\frac{1}{3(x+\sqrt{2})}$
(16) $\frac{1}{4(4x-1)}$
(17) $\ln(\frac{x+1}{x+2})$
(18) $\frac{1}{4}\ln(\frac{x-1}{x+3})$
(19) $\frac{1}{3}\ln(\frac{x-2}{x+1})$
(20) $\frac{1}{3}\ln(\frac{2x+1}{x+2})$
(21) $\frac{1}{5}\ln(\frac{3x-2}{x+1})$

Q21. (1) $\ln(x^2+4)$
(2) $\frac{1}{2}\ln(x^2+3)$
(3) $\frac{1}{2}\ln((x+3)^2+1)$
(4) $\frac{1}{8}\ln((2x-1)^2+4)$
(5) $\ln(x^2+6x+10) - 6\arctan(x+3)$
(6) $\frac{1}{2}\ln(x^2-8x+20) + 2\arctan(x-4)$
(7) $\frac{1}{2}\ln(x^2+3x+3) - \sqrt{3}\arctan(\frac{2x+3}{\sqrt{3}})$
(8) $-\frac{1}{2}\ln(x^2-4x+6) - \sqrt{2}\arctan(\frac{x-2}{\sqrt{2}})$
(9) $\frac{1}{3}\ln(x+2) - 2\sqrt{6}\arctan((\frac{\sqrt{3}x+2\sqrt{3}}{\sqrt{2}})$
(10) $\frac{1}{4}\ln(2x^2-10x+17) + \frac{5}{6}\arctan(\frac{2x-5}{3})$
(11) $\frac{1}{9}(\frac{2}{3x+2} + \ln(3x+2))$
(12) $\ln(x-2) - \frac{2}{x-2}$
(13) $\frac{1}{2}\ln(x^2+10x+25) + \frac{10}{x+5}$
(14) $\frac{1}{18}\ln(9x^2-12x+4) - \frac{4}{3x-2}$
(15) $\frac{1}{3}\ln(x+\sqrt{2}) + \frac{1}{3(x+\sqrt{2})}$
(16) $\frac{1}{16}\left(\frac{1}{4x-1} - \ln(1-4x)\right)$
(17) $\ln\frac{(x+1)^2}{x+2}$
(18) $\frac{1}{7}\ln\frac{(x-1)^2}{x+3}$
(19) $\frac{1}{3}\ln\frac{(x-2)^2}{x+1}$
(20) $\frac{1}{6}\ln\frac{(x+2)^4}{2x+1}$
(21) $\frac{2}{15}\ln(3x-2) + \frac{1}{5}\ln(x+1)$

Q22. (1) $\frac{1}{2}(x - 2\arctan\frac{x}{2})$
(2) $x^2 + \ln(x^2 + 3)$
(3) $\frac{3}{2}x^2 - x + 2\arctan(x + 3)$
(4) $\frac{3}{2}x^2 - x + \ln(x^2 + 6x + 10) + \arctan(x + 3)$
(5) $\frac{x^3}{3} + x + 3\arctan(\frac{x-4}{2})$
(6) $\frac{1}{3}x^3 - 20x - 50\ln(x^2 - 8x + 20)$
(7) $x^2 + 2x$
(8) $-x^2 + 3x - 9\sqrt{2}\arctan(\frac{x-2}{2})$
(9) $x - 2\ln(3x^2 + 12x + 14)$
(10) $x + \arctan(5 - x)$
(11) $\frac{1}{3}\ln(3x + 2) - \frac{1}{9x+6}$
(12) $\frac{1}{2}x^2 - x + \frac{1}{2} - \frac{1}{x-1}$
(13) $x - \frac{50}{x+5} - 10\ln(x + 5)$
(14) $\frac{1}{9}(3x + \frac{4}{3x-2} - 2)$
(15) $\ln(x + 1) + 2\ln(x - 2)$
(16) $2\ln(10 - 5x) - \frac{1}{2}\ln(10x + 5)$
(17) $\ln(\frac{3(x-2)^2}{x+1})$
(18) $x + \ln\frac{x+1}{(x+2)^2}$
(19) $\frac{2}{3}\ln(x + 2) + \frac{1}{3}\ln(x - 1)$
(20) $\frac{3}{4}x^2 - \frac{9}{4}x - \frac{3}{8}\ln(2x + 1) + 6\ln(x + 2)$
(21) $5x - \frac{2}{3}\ln(2 - 3x) - \ln(x + 1)$

25.4 More trigonometric substitutions

Questions 25.8.

Q23. (1) $\cos(x) + \ln(\tan(\frac{x}{2}))$
(2) $-\arctan(\cos x)$
(3) $\frac{\sqrt{2}}{4}\ln\left|\frac{\cos x-\sqrt{2}}{\cos x+\sqrt{2}}\right|$

Q24. (1) $\ln(\frac{1+\sin x}{\cos x})$
(2) $\ln(\frac{1+\sin x}{\cos x}) - \sin x$
(3) $\frac{\sqrt{2}}{4}\ln\left|\frac{\sin x+\sqrt{2}}{\sqrt{2}-\sin x}\right|$
(4) $\arctan(\sin(x))$

Q25. (1) $\frac{1}{\sqrt{2}}\arctan(\sqrt{2}\tan x)$
(2) $\frac{1}{\sqrt{2}}\arctan(\frac{1}{\sqrt{2}}\tan x)$
(3) $\frac{1}{6}\arctan(\frac{3}{2}\tan x)$
(4) $\frac{1}{6}\arctan(\frac{2}{3}\tan x)$
(5) $-\frac{1}{3}\arctan(3\cot x)$
(6) $\frac{1}{2}\arctan(2\tan x)$
(7) $\frac{1}{2\sqrt{3}}\arctan(\frac{\sqrt{3}}{2}\tan x)$
(8) $\frac{1}{\sqrt{2}}\arctan(\frac{1}{\sqrt{2}}\tan x)$
(9) $\frac{1}{\sqrt{3}}\arctan(\sqrt{3}\tan x)$
(10) $\frac{1}{4\sqrt{3}}\arctan(\frac{4}{\sqrt{3}}\tan x)$

Q26. (1) $\ln\left|\tan\left(\frac{x}{2}\right)\right|$
(2) $-\frac{\arctan(2\cos(\frac{x}{2}))}{\sin(\frac{x}{2})+\cos(\frac{x}{2})}$
(3) $\frac{2\sin(\frac{x}{2})}{9\cos(\frac{x}{2})-3\sin(\frac{x}{2})}$
(4) $\frac{2\sin(\frac{x}{2})}{9\cos(\frac{x}{2})-6\sin(\frac{x}{2})}$
(5) $\ln(2\cot(\frac{x}{2}) - 1)$
(6) $\frac{1}{3}\ln\frac{\sin(\frac{x}{2})}{\sin(\frac{x}{2})+3\cos(\frac{x}{2})}$
(7) $-\frac{2}{3}\arctan(3\cot(\frac{x}{2}))$
(8) $\frac{2\sin(\frac{x}{2})}{15\sin(\frac{x}{2})+9\cos(\frac{x}{2})}$
(9) $\frac{1}{2\sqrt{2}}\arctan(\frac{1}{\sqrt{2}}\tan(\frac{x}{2}))$
(10) $\frac{2}{5}\arctan(5\tan(\frac{x}{2}))$
(11) $\arctan(2\tan(\frac{x}{2}))$
(12) $\frac{1}{1+\cot(\frac{x}{2})}$
(13) $\frac{1}{2}\ln(\frac{2-\tan(\frac{x}{2})}{2+\tan(\frac{x}{2})})$
(14) $\ln(\frac{1-\cos x}{2+2\sin^2 x})$

25.5 Differential equations

Questions 25.9.

Q27. –

Q28. yes

Q29. (1) $y = \frac{1}{2}\arccos(C - 2\ln x)$
(2) $y = \ln(C - 3e^{-x})$
(3) $y = (\frac{1}{6}x^3 + C)^2$
(4) $y = \arccos k\csc x$
(5) $y = 3 - \frac{k}{\sqrt{2x+1}}$
(6) $y = \sqrt{\frac{1}{C-\arcsin x}}$
(7) $y = \tan(\frac{1}{2}x + \frac{1}{4}\sin 2x + C)$
(8) $y = \arctan(x + \frac{1}{3}x^3 + C)$

Q30. (1) $\ln y = \ln x^2$
(2) $y = \arctan(\frac{1}{2\cos x-1})$
(3) $y = \frac{1}{3-2\sqrt{x}}$
(4) $y = \ln(\frac{1}{1+e^{-2x}})$; $y(0) = -\ln 2$

Q31. $y^2 = x$

25.6 Mixed exercise

Questions 25.10.

Q32. (1) by parts
(2) by parts
(3) $\sin 6x$ formula
(4) $\cos 6x$ formula

Q33. $\frac{26}{15}$

Q34. $A=-1, B=2$,
$p=8, q=9$

Q35. (1) $A=-1, B=1, C=-2$
(2) $-3+6\ln 2$

Q36. (1) $\frac{4}{15}$
(2) $3-\ln 4$
(3) $\frac{1}{2}(4-\pi)$
(4) $\frac{15}{4}$
(5) $\pi+\frac{2}{\sqrt{3}}$
(6) $\frac{\pi}{3}$
(7) $\sqrt{3}(1-\ln 2)-\frac{\pi}{3}$

Q37. $2(1-\sqrt{x})-2\ln|1-\sqrt{x}|+c$

Q38. $\frac{1}{13}e^{2x}(2\sin 3x-3\cos 3x)+c$

Q39. (1) $\frac{1}{3}\ln(x^3-3x^2+6)+C$
(2) $\frac{1}{4}\tan^4 x+C$
(3) $-\frac{2}{3}(1-x)^{\frac{3}{2}}+\frac{4}{5}(1-x)^{\frac{5}{2}}-\frac{2}{7}(1-x)^{\frac{7}{2}}+C$
(4) $\frac{11}{24}$
(5) $\frac{1}{4}(\sin 2x-2(x+1)\cos 2x)+C$
(6) $2\ln^2 2-4\ln 2+2$
(7) $\frac{1}{2}e^x(\sin x+\cos x)+C$
(8) $\frac{\pi}{4}$
(9) $x\tan x+\ln\cos x$

Q40. 0

Q41. $\frac{26}{15}$

Q42. $\frac{1}{3}(2-\sqrt{2})$

PART VII

VECTORS

CHAPTER 26

INTRODUCTION TO VECTORS

QUESTIONS 26.1.

26.1 BASICS

Q1. (1) $\begin{pmatrix}10\\-11\\2\end{pmatrix}$ (2) $\pm\begin{pmatrix}\frac{2}{3}\\-\frac{11}{15}\\\frac{2}{15}\end{pmatrix}$ (3) $\pm\begin{pmatrix}\frac{14}{3}\\-\frac{77}{15}\\\frac{14}{15}\end{pmatrix}$

Q2. (1) $\begin{pmatrix}12\\-4\\3\end{pmatrix}$, (2) $\pm\begin{pmatrix}\frac{12}{13}\\-\frac{4}{13}\\\frac{3}{13}\end{pmatrix}$ (3) $\pm\begin{pmatrix}\frac{60}{13}\\-\frac{20}{13}\\\frac{15}{13}\end{pmatrix}$

Q3. (1) $(-5,6,-6)$ (2) $(1.5,2.5,2.5)$

Q4. (1) $\begin{pmatrix}12\\-12\\21\end{pmatrix}$ (2) $(6,-5,11)$ (4) $33\sqrt{2}$
(3) $(5,-4,12)$ (5) no

26.2 SCALAR PRODUCT

QUESTIONS 26.2.

Q5. (1) $\vec{AB}=\begin{pmatrix}2\\4\\4\end{pmatrix}$, $\vec{AC}=\begin{pmatrix}-2\\9\\-6\end{pmatrix}$ (2) $83.0°(1.45)$
(3) 32.8

Q6. (1) $\vec{AB} = \begin{pmatrix} -10 \\ 5 \\ -10 \end{pmatrix}$, $\vec{AC} = \begin{pmatrix} 3 \\ -2 \\ -6 \end{pmatrix}$
(2) $79.0°(1.38)$
(3) 51.5

Q7. (1) $\vec{AB} = \begin{pmatrix} -6 \\ 10 \\ 4 \end{pmatrix}$, $\vec{AC} = \begin{pmatrix} 9 \\ -15 \\ -6 \end{pmatrix}$
(2) $180°(\pi)$
(3) 0

Q8. $A \approx 78.2°(1.36), B \approx 63.2°(1.10), C \approx 38.6°(0.673)$

Q9. 1.5

Q10. 1.5 or -1

Q11. $c = \frac{1}{6}, d = \frac{4}{3}$

Q12. $\vec{u} = \pm(\frac{8}{3}\vec{i} + \frac{4}{3}\vec{j} - \frac{8}{3}\vec{k})$

Q13. ± 5

Q14. (1) 15
(2) $72.9°(1.27)$

Q15. $x = -1, y = 2$

CHAPTER 27

STRAIGHT LINES

Task

- 1 mile West, 7 miles South
- 21 miles East, 51 miles South
- 45 miles East, 100 miles South
- 15 miles West, 21 miles North
- 1^{st} of June at 02.30hrs, 4.5 miles South

27.1 EQUATIONS OF STRAIGHT LINES

QUESTIONS 27.1.

Q1. (i) $(-12, 4, 22)$
(ii) 2
(iii) (1) yes
(2) no
(3) no
(iv) $\lambda = 1, p = 0, q = 6$
(v) (1) $\begin{cases} x = 0 + 3\lambda \\ y = 0 - 1\lambda \\ z = 6 - 4\lambda \end{cases}$
(2) $\begin{cases} x = 9 + 6\lambda \\ y = -3 - 2\lambda \\ z = -6 - 8\lambda \end{cases}$
(3) $\begin{cases} x = 6 + 2\lambda \\ y = -2 - \frac{2}{3}\lambda \\ z = -2 - \frac{8}{3}\lambda \end{cases}$

Q2. (i) $y = \frac{1}{2}x + 1$ (ii) $y = -\frac{3}{2}x + \frac{7}{2}$ (iii) $y = -\frac{2}{3}x - \frac{5}{3}$ (iv) $y = -\frac{2}{3}x - \frac{8}{3}$

Q3. (i) $\begin{cases} x = 0 + \lambda \\ y = -3 + 2\lambda \end{cases}$ (ii) $\begin{cases} x = 0 + 4\lambda \\ y = -0.5 + 3\lambda \end{cases}$ (iii) $\begin{cases} x = 0 + 3\lambda \\ y = 1 - 2\lambda \end{cases}$ (iv) $\begin{cases} x = -2 + 5\lambda \\ y = 0 + 2\lambda \end{cases}$

Q4. (i)

λ	-2	-1	0	1	2
point	$(-4, -1)$	$(-2, 0)$	$(0, 1)$	$(2, 2)$	$(4, 3)$

(ii) *(sketch of $y = \frac{1}{2}x + 1$)*
(iii) $2\vec{i} + \vec{j}$

Q5. (i) e.g.$(5, -1, 2), (3, 1, 3), (1, 3, 4), (-1, 5, 5), (-3, 7, 6)$
(ii) $\begin{cases} x = 5 - 2t \\ y = -1 + 2t \\ z = 2 + t \end{cases}$
(iii) $t = \frac{x-5}{-2} = \frac{y+1}{2} = \frac{z-2}{1}$
(iv) $\frac{x-(5)}{(-2)} = \frac{y-(-1)}{(2)} = \frac{z-(2)}{(1)}$

Questions 27.2.

Q6. (1) $\frac{x-1}{1} = \frac{y+3}{2} = \frac{z+6}{5}$

(2) $\frac{x+3}{-2} = \frac{y-2}{1} = \frac{z-1}{-1}$

(3) $\frac{x-2}{-1} = \frac{y+1}{2} = \frac{z+5}{-3}$

(4) $\frac{x-3}{3} = \frac{z-2}{-1}, y = -1$

(5) $x - 3 = y + 1 = 1 - z$

(6) $\frac{x-3}{2} = \frac{y}{5}, z = -2$

(7) $x = 0, z = 1, y \in \mathbb{R}$

Q7. (1) $\vec{r} = \begin{pmatrix} 2 \\ 1 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$

(2) $\vec{r} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$

(3) $\vec{r} = \begin{pmatrix} -1 \\ -2 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

(4) $\vec{r} = \begin{pmatrix} -1.5 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$

(5) $\vec{r} = \begin{pmatrix} -4 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix}$

Q8. (i) $\frac{x-1}{4} = \frac{y-9}{5}, z = 5$

(ii) $\frac{x-3}{4} = \frac{y-5}{5}, z = 1$

Q9. (i) $\frac{x-6}{1} = \frac{y-2}{3} = \frac{z-8}{-4}$

(ii) $\frac{x-4}{1} = \frac{y-3}{3} = \frac{z-2}{-4}$

Q10. (i) $\frac{x-5}{5} = \frac{y-8}{9} = \frac{z-2}{6}$

(ii) $\frac{x-1}{5} = \frac{y-2}{9} = \frac{z-2}{6}$

27.2 Intersections of straight lines

Questions 27.3.

Q11. (i)

$$l_1 : \begin{cases} x = 10 - 6t \\ y = -7 + 3t \\ z = -21 + 12t \end{cases} \qquad l_4 : \begin{cases} x = -1 - \mu \\ y = 3 + 2\mu \\ z = -14 - 3\mu \end{cases}$$

$$l_2 : \begin{cases} x = -4 + 6s \\ y = -3s \\ z = 7 - 12s \end{cases} \qquad l_5 : \begin{cases} x = -4 - 2p \\ y = p \\ z = 7 \end{cases}$$

$$l_3 : \begin{cases} x = -10 + 4\lambda \\ y = 3 - 2\lambda \\ z = -19 - 8\lambda \end{cases} \qquad l_6 : \begin{cases} x = -11 + 7q \\ y = 5 - 5q \\ z = 16 - 9q \end{cases}$$

(ii)

?	l_1	l_2	l_3	l_4	l_5
l_6	$(-4, 0, 7)$	$(-4, 0, 7)$	skew	$(3, -5, -2)$	$(-4, 0, 7)$
l_5	$(-4, 0, 7)$	$(-4, 0, 7)$	$(-23, 9.5, 7)$	skew	
l_4	$(2, -3, -5)$	$(2, -3, -5)$	skew		
l_3	parallel	parallel			
l_2	coincident				

Q12. (1) $(-7, 4, 1)$, angle: $19.4°(0.339)$

(2) no intersection, (angle: $68.1°(1.19)$)

(3) no intersection, (angle: $19.5°(0.340)$)

(4) no intersection, (angle: $90°(\pi)$)

(5) $(-5, 3, -2)$, angle: $40.2°(0.702)$

chapter 28

Matrices

28.1 Basics

28.2 Multiplication of matrices

Questions 28.1.

Q1. $AB = \begin{pmatrix} -6 & -1 & -8 \\ -9 & 4 & -10 \end{pmatrix}$

Q2. (i) $M^2 = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$, $M^3 = \begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix}$, $M^4 = \begin{pmatrix} 1 & -4 \\ 0 & 1 \end{pmatrix}$,

(ii) $M^n = \begin{pmatrix} 1 & -n \\ 0 & 1 \end{pmatrix}$

(iii) —

Q3. (i) $\begin{pmatrix} 14 & 3 \\ 2 & 2 \end{pmatrix}$

(ii) $\begin{pmatrix} 14 & -1 \\ -6 & 2 \end{pmatrix}$

(iii) $\begin{pmatrix} -7 & 4 \\ 4 & -1 \end{pmatrix}$

(iv) $\begin{pmatrix} -9 & -2 \\ 0 & 1 \end{pmatrix}$

(v) $\begin{pmatrix} -25 & 13 \\ -2 & 5 \end{pmatrix}$

(vi) $\begin{pmatrix} -25 & 13 \\ -2 & 5 \end{pmatrix}$

Q4. (i) $\begin{pmatrix} 19 & 1 \\ -9 & -3 \end{pmatrix}$

(ii) $\begin{pmatrix} 16 & 12 \\ 4 & 0 \end{pmatrix}$

(iii) $\begin{pmatrix} 9 & 16 \\ 18 & 28 \end{pmatrix}$

(iv) $\begin{pmatrix} 0 & 0 & -10 \\ -4 & 0 & 3 \\ 5 & 0 & -5 \end{pmatrix}$

(v) $\begin{pmatrix} 12 & -6 & 9 \\ -6 & 0 & 3 \\ 9 & 3 & -6 \end{pmatrix}$

(vi) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$

(vii) $\begin{pmatrix} -2 & 3 \\ -7 & 5 \end{pmatrix}$

(viii) $\begin{pmatrix} -1 & 5 & -1 \\ -73 & 8 & 80 \\ 2 & 4 & -4 \end{pmatrix}$

Q5. A - anticlockwise rotation by 90° about the origin,
B - anticlockwise rotation by 45° about the origin,
C - reflection in the x-axis,
D - reflection in the y-axis,
E - reflection in the line $y = x$

Q6. $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Q7. $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

28.3 Determinant of a matrix, inverse matrix

Questions 28.2.

Q8. (i) $AB = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

(ii) $A^{-1} = \begin{pmatrix} 0 & \frac{1}{2} \\ 1 & \frac{3}{2} \end{pmatrix}$

Q9. (i) $AB = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}$.

(ii) $A^{-1} = \begin{pmatrix} 0.6 & 0.4 & -0.2 \\ -0.8 & -0.2 & 0.6 \\ -1 & -1 & 1 \end{pmatrix}$.

Q10. —

Q11. —

Q12. —

Questions 28.3.

Q13. (i) (a) $\begin{pmatrix} -2 & 1 \\ 1.5 & -0.5 \end{pmatrix}$

(b) $\begin{pmatrix} -3 & 4 \\ -2 & 3 \end{pmatrix}$

(c) $\begin{pmatrix} -0.25 & 1.25 \\ 0.5 & -1.5 \end{pmatrix}$

(d) $\begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix}$

(e) $\frac{1}{6}\begin{pmatrix} -4 & 3 \\ 2 & -3 \end{pmatrix}$

(f) $\begin{pmatrix} -2 & -3 \\ 3 & 5 \end{pmatrix}$

(g) $\frac{1}{4}\begin{pmatrix} 3 & -1 \\ -5 & 3 \end{pmatrix}$

(h) $\frac{1}{5}\begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$

(i) $\frac{1}{5}\begin{pmatrix} -3 & -7 \\ -2 & -3 \end{pmatrix}$

(j) no inverse

(k) $\frac{1}{3}\begin{pmatrix} -6 & -3 \\ -5 & -2 \end{pmatrix}$

(l) $\frac{1}{15}\begin{pmatrix} 9 & -12 \\ 6 & 2 \end{pmatrix}$

(ii) —

Questions 28.4.

Q14. —

Q15. (i) -2

(ii) -6

(iii) -8

(iv) 0 (singular)

(v) -23

(vi) 0 (singular)

Q16. (i) $\det A = 4, \det B = -2, \det C = -2$
(ii) $\det(AC^{-1}B) = (4)(-\frac{1}{2})(-2) = 4.$
(iii) $\frac{1}{4}\begin{pmatrix} -1 & 3 \\ -3 & 5 \end{pmatrix}$
(iv) $\frac{1}{-2}\begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}$
(v) $\frac{1}{8}\begin{pmatrix} 15 & -29 \\ 23 & -45 \end{pmatrix}$
(vi) $\frac{1}{-4}\begin{pmatrix} 6 & 7 \\ 8 & 9 \end{pmatrix}$

Questions 28.5.

Q17. -2

Q18. (i) —
(ii) $\begin{pmatrix} a^2 - b^2 & 2ab \\ -2ab & a^2 - b^2 \end{pmatrix}$
(iii) —

Q19. 24 **Q20.** 5 **Q21.** -41

Q22. (i) $k = -4,\ s = 3.$
(ii) $m = 40, t = -39$
(iii) $k = -6, s = 5, m = 156, t = -155$

Q23. (i) $-1, 1, 2$
(ii) $\lambda = -1 \Rightarrow \vec{v} = \begin{pmatrix} t \\ -2.5t \\ 0.5t \end{pmatrix}, t \in \mathbb{R};$
$\lambda = 1 \Rightarrow \vec{v} = \begin{pmatrix} t \\ -2t \\ -t \end{pmatrix}, t \in \mathbb{R};$
$\lambda = 2 \Rightarrow \vec{v} = \begin{pmatrix} t \\ -t \\ -t \end{pmatrix}, t \in \mathbb{R}$

Q24. (i) —
(ii) 1

Q25. (i) incorrect; $(A + B)^2 = A^2 + B^2 + AB + BA$
(ii) incorrect; $ADB = C \Leftrightarrow D = A^{-1}CB^{-1}$
(iii) correct
(iv) incorrect; $AB = I \Rightarrow B = A^{-1}$
(v) incorrect; $(A + 2I)^2 = A^2 + 4A + 4I$
(vi) correct
(vii) correct
(viii) correct when order of A is odd, incorrect otherwise
(ix) correct when order of A is even, incorrect otherwise

28.4 Solving simultaneous equations - intersections of the planes

Questions 28.6.

Q26. $x = \frac{1}{3}, y = -\frac{4}{3}, z = 0$

Q27. $x = -3, y = 18, z = 31$

chapter 29

Planes in three-dimensional space

29.1 Intersections of the planes

Gaussian elimination (row reduction)

Questions 29.1.

Q1. (1) line $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}\frac{4}{7}\\\frac{1}{7}\\0\end{pmatrix} + t\begin{pmatrix}-2\\17\\7\end{pmatrix}$

(2) line $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}2\\1\\0\end{pmatrix} + t\begin{pmatrix}1\\1\\1\end{pmatrix}$

(3) line $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}-\frac{10}{7}\\-\frac{2}{7}\\0\end{pmatrix} + t\begin{pmatrix}1\\3\\7\end{pmatrix}$

(4) no intersection

(5) point $(2, 2, -1)$

(6) point $(2.5, -1, 0.5)$

(7) point $(-2, -5.5, -2.5)$

(8) line $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}2\\3\\0\end{pmatrix} + t\begin{pmatrix}3\\1\\1\end{pmatrix}$

(9) line $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}5\\-4\\0\end{pmatrix} + t\begin{pmatrix}0\\1\\1\end{pmatrix}$

(10) line $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}-3\\0.5\\0\end{pmatrix} + t\begin{pmatrix}-5\\6\\2\end{pmatrix}$

(11) line $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}5\\0\\-7\end{pmatrix} + t\begin{pmatrix}1\\1\\0\end{pmatrix}$

(12) point $(-6, -10, 3)$

(13) point $(2, 1, -1)$

(14) point $(-1, -4, 2)$

(15) point $(-2, 1, -1)$

(16) point $(1, 1, -2)$

(17) point $(5, -2, 1)$

(18) point $(4, 3, -2)$

(19) plane $x - 2y - 3z = 2$

(20) no intersection

(21) line $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}\frac{13}{29}\\\frac{1}{29}\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\1\end{pmatrix}$

(22) no intersection

Q2. (1) $(2, -1, -1)$ (2) $(-2, -1, 5)$ (3) $(-4, 3, 5)$ (4) whole line

29.2 Vector product

29.3 Properties of scalar and vector product

Questions 29.2.

Q3. (i) $-60\vec{i} + 4\vec{j} + 26\vec{k}$ (ii) $-30x + 2y + 13z = 113$ (iii) $\sqrt{1073}$

Q4. (i) $-50\vec{i} - 90\vec{j} + 5\vec{k}$ (ii) $-10x - 18y + z = 16$ (iii) $\frac{25}{2}\sqrt{17}$

Q5. (i) $0\vec{i} + 0\vec{j} + 0\vec{k}$ (ii) no unique plane (iii) 0

Q6. (i) $(3, 1, -1)$ (ii) $9\vec{i} + 23\vec{j} + 1\vec{k}$ (iii) $\sqrt{611}$

Q7. (i) $(2, 7, 6)$ (ii) $-9\vec{i} - 5\vec{j} + 17\vec{k}$ (iii) $\sqrt{395}$

Q8. $7x + 4y + 5z = -3$

Q9. $-2x - 11y + 7z = 16$

Q10. $4x - 3z = 11$

Q11. (i) $x + 4y - 4z = 1$

(ii) $\vec{i} + 4\vec{j} - 4\vec{k}$

(iii) $\vec{r} = \begin{pmatrix}5\\-4\\-3\end{pmatrix} + \lambda\begin{pmatrix}1\\4\\-4\end{pmatrix}$

(iv) —

(v) $\sqrt{33}$

(vi) 33

Q12. (i) —

(ii) $3x + 2y - z = 13$

(iii) $3\vec{i} + 2\vec{j} - \vec{k}$

(iv) $(3, 1, -2)$

(v) $\vec{r} = \begin{pmatrix}3\\1\\-2\end{pmatrix} + \lambda\begin{pmatrix}3\\2\\-1\end{pmatrix}$

(vi) $(2 - 3t, -1 - 2t, 4 + t)$

(vii) $\pm\sqrt{2}$

(viii) $28\sqrt{2}$

Q13. (1) — (2) — (3) 0 (4) 90° (5) — (6) — (7) —

Q14. whole line

Q15. $9x + 21y + 2z = 33$

Q16. $x + z + 3 = 0$

Q17. $25.1°(0.438), (0, -\frac{11}{3}, 0)$

29.4 Alternate forms of equations of planes

Questions 29.3.

Q18. (1) $-4x + 6y + 5z + 12 = 0$ (2) $x - y + 3z + 5 = 0$ (3) $y + z = 2$

Q19. a number of possibilities

Q20. a number of possibilities

29.5 3D distances

Distance between a point and a plane

Questions 29.4.

Q21. (i) $2\vec{i}-5\vec{j}+\vec{k}$

(ii) $\vec{r}=\begin{pmatrix}-4\\9\\-4\end{pmatrix}+t\begin{pmatrix}2\\-5\\1\end{pmatrix}$

(iii) $(0,-1,-2)$

(iv) $2\sqrt{30}$

Q22. $\frac{5}{2}\sqrt{62}$

Q23. 8

Q24. $2\sqrt{10}$

Distance between two (parallel) planes

Q25. (i) lots of them

(ii) $2\sqrt{14}$

(iii) $2\sqrt{14}$

Q26. $3\sqrt{51}$

Q27. 13

Q28. $\frac{6}{5}\sqrt{2}$

Distance between a point and a line

Q29. (1st method)

(i) $\begin{cases}x=3t\\y=7-5t\\z=-3.5+2t\end{cases}$

(ii) $\begin{pmatrix}3t\\7-5t\\-3.5+2t\end{pmatrix}$

(iii) $\begin{pmatrix}3t-1\\6-5t\\z=-2.5+2t\end{pmatrix}$

(iv) $0,(3,2,-1.5)$

(v) $\frac{1}{2}\sqrt{21}$

Q29. (2nd method)

(i) $3\vec{i}-5\vec{j}+2\vec{k}$

(ii) $3x-5y+2z=-4$

(iii) $(3,2,-1.5)$

(iv) $\frac{1}{2}\sqrt{21}$

Q29. (3rd method)

(i) $3\vec{i}-5\vec{j}+2\vec{k}$

(ii) e.g. $(0,7,-3.5)$

(iii) $3\vec{i}-5\vec{j}+2\vec{k}$

(iv) $\frac{1}{2}\sqrt{21}$

(v) $\frac{1}{2}\sqrt{21}$

Q30. $\frac{1}{2}\sqrt{11}$

Q31. $\sqrt{26}$

Distance between parallel lines

Q32. (i) $\begin{cases}x=2-2t\\y=5+3t\\z=6-t\end{cases}$

(ii) e.g. $(1,-1,-10)$

(iii) $\sqrt{293}$

(iv) $\sqrt{293}$

Q33. $\frac{1}{2}\sqrt{145}$

Q34. $\sqrt{30}$

Q35. $\sqrt{29}$

Distance between skew lines

Questions 29.5.

Q36. (1st method)

(i) $\begin{cases}x=-2+3t\\y=8+3t\\z=3+4t\end{cases}$

(ii) $\begin{cases}x=-4+s\\y=-7+2s\\z=2+3s\end{cases}$

(iii) $\begin{pmatrix}-2+3t\\8+3t\\3+4t\end{pmatrix}$

(iv) $\begin{pmatrix}-4+s\\-7+2s\\2+3s\end{pmatrix}$

(v) $\begin{pmatrix}-2+1s+-3t\\-15+2s+-3t\\-1+3s+-4t\end{pmatrix}$

(vi) $s=1,t=-1$

(vii) $2\sqrt{35}$

Q36. (2nd method)

(i) $\begin{pmatrix}3\\3\\4\end{pmatrix}$

(ii) $\begin{pmatrix}1\\2\\3\end{pmatrix}$

(iii) $(-2,8,3)$

(iv) $(-4,-7,2)$

(v) $\begin{pmatrix}1\\-5\\3\end{pmatrix}$

(vi) $x - 5y + 3z = -33$
(vii) —
(viii) —
(ix) $2\sqrt{35}$
(x) $2\sqrt{35}$

Q36. (3rd method)

(i) $\begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix}$
(ii) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$
(iii) $(-2, 8, 3)$
(iv) $(-4, -7, 2)$
(v) $\begin{pmatrix} 1 \\ -5 \\ 3 \end{pmatrix}$
(vi) $\begin{pmatrix} 2 \\ -10 \\ 6 \end{pmatrix}$
(vii) $2\sqrt{35}$

Q37. 4.04

Q38. $3\sqrt{11}$

29.6 Angles between lines, planes...

Questions 29.6.

Q39. 0° **Q40.** 72° **Q41.** 41° **Q42.** 65° **Q43.** 50.0° **Q44.** 36.2°

part VIII

Complex numbers

chapter 30

Introduction to complex numbers

30.1 Polar coordinates

Questions 30.1.

Q1. (1) $(4, \frac{\pi}{3})$
(2) $(2\sqrt{3}, \frac{\pi}{6})$
(3) $(3, \pi)$
(4) $(2, -\frac{\pi}{2})$
(5) $(5\sqrt{2}, \frac{3\pi}{4})$
(6) $(4\sqrt{3}, -\frac{2\pi}{3})$
(7) $(8, -\frac{\pi}{6})$
(8) $(3\sqrt{2}, -\frac{\pi}{4})$
(9) $(2.5\sqrt{2}, \frac{3\pi}{4})$
(10) $(6\sqrt{3}, \frac{5\pi}{6})$
(11) $(10/3, 0)$
(12) $(8\sqrt{2}, -\frac{3\pi}{4})$
(13) $(2\sqrt{3}, \frac{2\pi}{3})$
(14) $(4\sqrt{6}, -\frac{5\pi}{6})$
(15) $(\sqrt{5}, \frac{\pi}{2})$
(16) $(5, 0.927)$
(17) $(6.08, 0.165)$
(18) $(8.54, 2.78)$
(19) $(6.16, -0.409)$
(20) $(8.6, -2.19)$
(21) $(6.02, 1.65)$
(22) $(7.01, -0.0485)$
(23) $(5.73, -2.72)$
(24) $(0.734, 1.86)$
(25) $(13, 2.75)$
(26) $(11.8, -1.20)$
(27) $(17, -2.65)$

Q2. (1) $(3, 3\sqrt{3})$
(2) $(4, 4)$
(3) $(2\sqrt{3}, 2)$
(4) $(-3\sqrt{3}, 3)$
(5) $(-1, -\sqrt{3})$
(6) $(-1.2\sqrt{2}, 1.2\sqrt{2})$
(7) $(\frac{3}{2}, -\frac{3\sqrt{3}}{2})$
(8) $(-\frac{9}{5}, -\frac{9}{5})$
(9) $(-\sqrt{3}, 3)$
(10) $(3, -\sqrt{3})$
(11) $(\frac{3}{2}, -\frac{3}{2})$
(12) $(-6\sqrt{3}, -6)$
(13) $(-2.97, 6.07)$
(14) $(2, 16, 3.37)$
(15) $(-1.04, -2.27)$
(16) $(-0.801, -0.598)$
(17) $(7.17, 3.55)$
(18) $(1.70, 2.64)$
(19) $(8.86, -8.09)$
(20) $(-4.70, -8.03)$

30.2 Introducing complex numbers

Questions 30.2.

Q3. e.g. $a = b = -1$

Q4. e.g. $a = 1, b = -1$

Questions 30.3.

Q5. (1) $\pm 3i$
(2) $1 \pm 2i$
(3) $1 \pm 3i$
(4) $1 \pm i\sqrt{2}$
(5) $-3 \pm i\sqrt{2}$
(6) $\frac{1}{2} \pm \frac{3}{2}i\sqrt{3}$
(7) $-1 \pm i\sqrt{6}$
(8) $\pm\frac{4}{3}i$
(9) $\pm i\sqrt{3}$
(10) $-\frac{1}{2} \pm i\frac{\sqrt{5}}{2}$
(11) $\frac{1}{3} \pm \frac{2}{3}i\sqrt{5}$
(12) $-\frac{1}{2} \pm \frac{1}{2}i$
(13) $\frac{1}{3} \pm i\frac{\sqrt{2}}{3}$
(14) $-1, \pm 2i$
(15) $3, \pm\frac{1}{\sqrt{2}}i$
(16) $\pm i, \pm 2i$
(17) $\pm 1, \pm i$
(18) $-1, \frac{1}{2} \pm i\frac{\sqrt{3}}{2}$

30.3 Algebra of complex numbers

Questions 30.4.

Q6. (i) $9 + 7i$
(ii) $-1 + 7i$
(iii) $-8 + i$
(iv) $8 + 14i$
(v) $-26 + 7i$
(vi) $-15 - 8i$

Q7. (i) $-i$
(ii) 1
(iii) i
(iv) -1
(v) $-i$
(vi) 1
(vii) $2i$
(viii) $-2 + 2i$
(ix) -4
(x) 16
(xi) $16 + 16i$
(xii) 0

Q8. $a = -3, b = 2$

30.4 Conjugate numbers. Polar form of a complex number

Questions 30.5.

Q9.

z	$\lvert z\rvert$	z^*	zz^*
$3 - 4i$	5	$3 + 4i$	25
$12i - 5$	13	$-12i - 5$	169
$2\sqrt{3} + 6i$	$4\sqrt{3}$	$2\sqrt{3} - 6i$	48
$4 - 4i$	$4\sqrt{2}$	$4 + 4i$	32
$-1 + 3i$	$\sqrt{10}$	$-1 - 3i$	10

Questions 30.6.

Q10. (i) $0.3 + 1.1i$
(ii) $-1 - i$
(iii) $\frac{4}{13} + \frac{7}{13}i$
(iv) $\frac{3}{13} - \frac{11}{13}i$
(v) $-\frac{1}{2} + \frac{1}{2}i$
(vi) $0.8 - 1.4i$

Q11.

z in cartesian form	z in polar form	z^* in cartesian form	z^* in polar form
$4 + 4i$	$4\sqrt{2}cis\frac{\pi}{4}$	$4 - 4i$	$4\sqrt{2}cis(-\frac{\pi}{4})$
$-\sqrt{3} + 3i$	$2\sqrt{3}cis\frac{2\pi}{3}$	$-\sqrt{3} - 3i$	$2\sqrt{3}cis(-\frac{2\pi}{3})$
$\sqrt{3} - i$	$2cis(-\frac{\pi}{6})$	$\sqrt{3} + i$	$2cis\frac{\pi}{6}$
$-\sqrt{3} + i$	$2cis\frac{5\pi}{6}$	$-\sqrt{3} - i$	$2cis(-\frac{5\pi}{6})$
$-2 - 2i\sqrt{3}$	$4cis(-\frac{2\pi}{3})$	$-2 + 2i\sqrt{3}$	$4cis\frac{2\pi}{3}$
$-3i$	$3cis(-\frac{\pi}{2})$	$3i$	$3cis\frac{\pi}{2}$

Questions 30.7.

Q12.

	z (polar)	v (polar)	zv (cartesian)	zv (polar)
1	$4cis\frac{\pi}{6}$	$3cis\frac{\pi}{2}$	$-6+6i\sqrt{3}$	$12cis\frac{2\pi}{3}$
2	$4\sqrt{3}cis\frac{\pi}{6}$	$2cis\frac{2\pi}{3}$	$-12+4i\sqrt{3}$	$8\sqrt{3}cis\frac{5\pi}{6}$
3	$5\sqrt{2}cis\frac{3\pi}{4}$	$2cis(-\frac{\pi}{2})$	$10+10i$	$10\sqrt{2}cis\frac{\pi}{4}$
4	$2\sqrt{3}cis(-\frac{5\pi}{6})$	$4cis\frac{2\pi}{3}$	$12-4i\sqrt{3}$	$8\sqrt{3}cis(-\frac{\pi}{6})$
5	$2\sqrt{2}cis(-\frac{3\pi}{4})$	$3\sqrt{2}cis(-\frac{3\pi}{4})$	$12i$	$12cis\frac{\pi}{2}$
6	$3\sqrt{2}cis\frac{\pi}{4}$	$\sqrt{2}cis\frac{3\pi}{4}$	-6	$6cis\pi$
7	$2cis(-\frac{2\pi}{3})$	$2cis(-\frac{\pi}{3})$	-4	$4cis\pi$
8	$\sqrt{2}cis\frac{\pi}{4}$	$2cis(-\frac{\pi}{6})$	$(\sqrt{3}+1)+i(\sqrt{3}-1)$	$2\sqrt{2}cis\frac{\pi}{12}$

Questions 30.8.

Q13.

	z (polar)	v (polar)	$\frac{z}{v}$ (cartesian)	$\frac{z}{v}$ (polar)
1	$4cis\frac{\pi}{6}$	$3cis\frac{\pi}{2}$	$\frac{2}{3}-(2\sqrt{3})/3i$	$\frac{4}{3}cis(-\frac{\pi}{3})$
2	$4\sqrt{3}cis\frac{\pi}{6}$	$2cis\frac{2\pi}{3}$	$-2i\sqrt{3}$	$2\sqrt{3}cis(-\frac{\pi}{2})$
3	$5\sqrt{2}cis\frac{3\pi}{4}$	$2cis(-\frac{\pi}{2})$	$-\frac{5}{2}-\frac{5}{2}i$	$(5\sqrt{2})/2cis(-\frac{3\pi}{4})$
4	$2\sqrt{3}cis(-\frac{5\pi}{6})$	$4cis\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}i$	$\frac{\sqrt{3}}{2}cis(\frac{\pi}{2})$
5	$2\sqrt{2}cis(-\frac{3\pi}{4})$	$3\sqrt{2}cis(-\frac{3\pi}{4})$	$\frac{2}{3}$	$\frac{2}{3}cis0$
6	$3\sqrt{2}cis\frac{\pi}{4}$	$\sqrt{2}cis\frac{3\pi}{4}$	$-3i$	$3cis(-\frac{\pi}{2})$
7	$2cis(-\frac{2\pi}{3})$	$2cis(-\frac{\pi}{3})$	$1/2+i\frac{\sqrt{3}}{2}$	$cis(-\frac{\pi}{3})$
8	$\sqrt{2}cis\frac{\pi}{4}$	$2cis(-\frac{\pi}{6})$	$(\sqrt{3}-1)/2+i(\sqrt{3}+1)/2$	$1/\sqrt{2}cis\frac{5\pi}{12}$

CHAPTER 31

De Moivre's theorem

Questions 31.1.

Q1.

	z^2	z^3	z (polar)	z^2 (polar)	z^3 (polar)
1	$8+8i\sqrt{3}$	$64i$	$4cis\frac{\pi}{6}$	$16cis\frac{\pi}{3}$	$64cis\frac{\pi}{2}$
2	-9	$-27i$	$3cis\frac{\pi}{2}$	$9cis\pi$	$27cis(-\frac{\pi}{2})$
3	$24+24i\sqrt{3}$	$192\sqrt{3}i$	$4\sqrt{3}cis\frac{\pi}{6}$	$48cis\frac{\pi}{3}$	$192\sqrt{3}cis\frac{\pi}{2}$
4	$-2-2i\sqrt{3}$	8	$2cis\frac{2\pi}{3}$	$4cis(-\frac{2\pi}{3})$	$8cis0$
5	$-50i$	$250+250i$	$5\sqrt{2}cis\frac{3\pi}{4}$	$50cis(-\frac{\pi}{2})$	$250\sqrt{2}cis\frac{\pi}{4}$
6	-4	$8i$	$2cis(-\frac{\pi}{2})$	$4cis()$	$8cis(\frac{\pi}{2})$
7	$6+6i\sqrt{3}$	$-24\sqrt{3}i$	$2\sqrt{3}cis(-\frac{5\pi}{6})$	$12cis(\frac{\pi}{3})$	$24\sqrt{3}cis(-\frac{\pi}{2})$
8	$-8-8i\sqrt{3}$	64	$4cis\frac{2\pi}{3}$	$16cis(-\frac{2\pi}{3})$	$64cis0$
9	$8i$	$16-16i$	$2\sqrt{2}cis(-\frac{3\pi}{4})$	$8cis\frac{\pi}{2}$	$16\sqrt{2}cis(-\frac{\pi}{4})$
10	$18i$	$-54-54i$	$3\sqrt{2}cis\frac{\pi}{4}$	$18cis\frac{\pi}{2}$	$54\sqrt{2}cis\frac{3\pi}{4}$
11	$-2i$	$2+2i$	$\sqrt{2}cis\frac{3\pi}{4}$	$2cis(-\frac{\pi}{2})$	$2\sqrt{2}cis\frac{\pi}{4}$
12	$-2+2i\sqrt{3}$	8	$2cis(-\frac{2\pi}{3})$	$4cis\frac{2\pi}{3}$	$8cis0$
13	$-2-2i\sqrt{3}$	-8	$2cis(-\frac{\pi}{3})$	$4cis(-\frac{2\pi}{3})$	$8cis\pi$
14	$2-2i\sqrt{3}$	-8	$2cis(-\frac{\pi}{6})$	$4cis(-\frac{\pi}{3})$	$8cis\pi$

Q2.

	z^4	z (polar)	z^4 (polar)
1	$-8+8i\sqrt{3}$	$2cis\frac{\pi}{6}$	$16cis\frac{2\pi}{3}$
2	16	$2cis\frac{\pi}{2}$	$16cis0$
3	$-8+8i\sqrt{3}$	$2cis\frac{2\pi}{3}$	$16cis\frac{2\pi}{3}$
4	-2500	$5\sqrt{2}cis\frac{3\pi}{4}$	$2500cis\pi$
5	81	$3cis(-\frac{\pi}{2})$	$81cis0$
6	$-72+72i\sqrt{3}$	$2\sqrt{3}cis(-\frac{5\pi}{6})$	$144cis\frac{2\pi}{3}$
7	-64	$2\sqrt{2}cis(-\frac{3\pi}{4})$	$64cis\pi$
8	-4	$\sqrt{2}cis\frac{\pi}{4}$	$4cis\pi$
9	-4	$\sqrt{2}cis\frac{3\pi}{4}$	$4cis\pi$
10	$-8-8i\sqrt{3}$	$2cis(-\frac{2\pi}{3})$	$16cis(-\frac{2\pi}{3})$
11	$-8+8i\sqrt{3}$	$2cis(-\frac{\pi}{3})$	$16cis\frac{2\pi}{3}$
12	$-8-8i\sqrt{3}$	$2cis(-\frac{\pi}{6})$	$16cis(-\frac{2\pi}{3})$

Questions 31.2.

Q3. (1) circle centre 0, radius 2

(2) circle centre 0, radius 5

(3) line $y = \sqrt{3}x, x > 0$

(4) line $y = \frac{x}{\sqrt{3}}, x < 0$

(5) circle centre 2, radius 2

(6) circle centre $-3i$, radius 2

(7) circle centre $2 - 3i$, radius 2

(8) line $y = \sqrt{3}(x-2), x > 2$

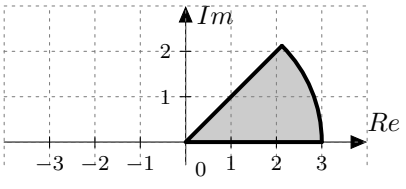
(9) line $y = 1, x < 0$

(10) line $x = 1$

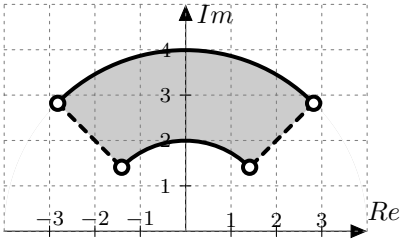
(11) line $y = -\frac{1}{2}x + \frac{3}{2}$

(12) line $y = \frac{4}{3}x - \frac{2}{3}$

(13)



(14)



(15) line $y = \frac{x}{\sqrt{3}}, x \neq 0$

(16) line $y = \sqrt{3}(x+1), x \neq -1$

(17) circle centre 4, radius 2

(18) circle centre $\frac{5}{3}$, radius $\frac{4}{3}$

Q4. $\pm(2+i)$

Q5. $\pm(3-i)$

Q6. (1) $|3z| \leq 9, \frac{\pi}{3} \leq \arg 3z < \frac{\pi}{2}$

(2) $|\frac{z}{2}| \leq \frac{3}{2}, \frac{\pi}{3} \leq \arg \frac{z}{2} < \frac{\pi}{2}$

(3) $|-z| \leq 3, -\frac{2\pi}{3} \leq \arg(-z) < -\frac{\pi}{2}$

(4) $|z^2| \leq 9, \frac{2\pi}{3} \leq \arg z^2 < \pi$

(5) $|z^3| \leq 27, \pi \leq \arg z < \frac{3\pi}{2}$

(6) $|\sqrt{z}| \leq \sqrt{3}, \frac{\pi}{6} \leq \arg z < \frac{\pi}{4}$

Q7. (1) $3 < |3z| \leq 6, -\frac{\pi}{3} \leq \arg 3z < \frac{\pi}{2}$

(2) $\frac{1}{2} < |\frac{z}{2}| \leq 1, -\frac{\pi}{3} \leq \arg \frac{z}{2} < \frac{\pi}{2}$

(3) $1 < |z| \leq 2, \frac{2\pi}{3} \leq \arg(-z) < \frac{3\pi}{2}$

(4) $1 < |z^2| \leq 4, -\frac{2\pi}{3} \leq \arg z^2 < \pi$

(5) $1 < |z^3| \leq 8, -\pi \leq \arg z^3 < \frac{3\pi}{2}$
i.e. no restrictions

(6) $1 < |\sqrt{z}| \leq \sqrt{2}, -\frac{\pi}{6} \leq \arg \sqrt{z} < \frac{\pi}{4}$

(7) $\frac{1}{2} \leq |\frac{1}{z}| < 1, -\frac{\pi}{2} < \arg \frac{1}{z} \leq \frac{\pi}{3}$

(8) $2 < |2z^2| \leq 8, -\frac{2\pi}{3} \leq \arg 2z^2 < \pi$

Q8. (1) $3 \leq |3z|, -\frac{\pi}{2} \leq \arg 3z < -\frac{\pi}{6}$

(2) $\frac{1}{2} \leq |\frac{z}{2}|, -\frac{\pi}{2} \leq \arg \frac{z}{2} < -\frac{\pi}{6}$

(3) $1 \leq |z|, \frac{\pi}{2} \leq \arg(-z) < \frac{5\pi}{6}$

(4) $1 \leq |z^2|, -\pi \leq \arg z^2 < -\frac{\pi}{3}$

(5) $1 \leq |z^3|, -3\frac{\pi}{2} \leq \arg z^3 < -\frac{\pi}{2}$
i.e. no restrictions

(6) $1 \leq |\sqrt{z}|, -\frac{\pi}{4} \leq \arg \sqrt{z} < -\frac{\pi}{12}$

(7) $1 \geq |\frac{1}{z}|, \frac{\pi}{6} < \arg \frac{1}{z} \leq \frac{\pi}{2}$

(8) $2 \leq |2z^2|, -\pi \leq \arg z < -\frac{\pi}{3}$

Q9. —

31.1 De Moivre's theorem and its generalization

Questions 31.3.

Q10. (i) $r = 2$

(ii) $\theta \in \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\}$

(iii) $z \in \{\sqrt{3}+i, -\sqrt{3}+i, -2i\}$

Q11. (i) ± 1 (ii) $1, -\frac{1}{2} \pm \frac{\sqrt{3}}{2}$ (iii) $\pm 1, \pm i$ (iv) $\pm 1, \pm(\frac{1}{2} \pm i\frac{\sqrt{3}}{2})$

Q12. (i) $z \in \{2, -1 \pm i\sqrt{3}\}$

(ii) $z \in \{-3, \frac{3}{2} \pm i\frac{3\sqrt{3}}{2}\}$

(iii) $z = \pm(\sqrt{2} \pm i\sqrt{2})$

(iv) $z \in \{-3\sqrt{2}, \frac{3\sqrt{2}}{2} \pm i\frac{3\sqrt{6}}{2}\}$

(v) $z \in \{\pm(\sqrt{3}+i), \pm(-1+i\sqrt{3})\}$

(vi) $z \in \{\pm(\frac{\sqrt{6}}{2} - i\frac{\sqrt{2}}{2}), \pm(\frac{\sqrt{2}}{2} + i\frac{\sqrt{6}}{2})\}$

(vii) $z \in \{\pm\frac{3}{2} + i\frac{\sqrt{3}}{2}, -i\sqrt{3}\}$

Q13. $\cos\frac{\pi}{12}=\frac{\sqrt{6}+\sqrt{2}}{4}, \sin\frac{\pi}{12}=\frac{\sqrt{6}-\sqrt{2}}{4}, \tan\frac{\pi}{12}=2-\sqrt{3}$

Q14. $\cos\frac{5\pi}{12}=\frac{\sqrt{6}-\sqrt{2}}{4}, \sin\frac{5\pi}{12}=\frac{\sqrt{6}+\sqrt{2}}{4}, \tan\frac{5\pi}{12}=2+\sqrt{3}$
$\cos\frac{7\pi}{12}=\frac{\sqrt{2}-\sqrt{6}}{4}, \sin\frac{7\pi}{12}=\frac{\sqrt{6}+\sqrt{2}}{4}, \tan\frac{7\pi}{12}=-2-\sqrt{3}$

Q15. (1) $a=5$ (2) $b=6$ (3) $c=5, d=2$

Q16. (1) $a=0, b=24\sqrt{3}$
(2) $-3+i\sqrt{3}, -2\sqrt{3}i$
(3) $\frac{\sqrt{3}}{2}+\frac{1}{2}i, \frac{\sqrt{3}}{2}+\frac{7}{2}i, -\sqrt{3}+2i$

Q17. modulus, argument:

(1) 5, $\frac{\pi}{2}$
(2) 3, 2
(3) e^2, 2
(4) $4e^2$, -1
(5) πe^2, π
(6) $\sqrt{3}$, 3
(7) 1, $\ln 2$
(8) 3, $\ln 3$
(9) 25, $-3\ln 5$

CHAPTER 32

CONJUGATE ROOTS

32.1 FUNDAMENTAL THEOREM OF ALGEBRA

QUESTIONS 32.1.

Q1. (1) $x=\pm 2i$
(2) $x\in\{-1, \pm i\}$
(3) $x\in\{-1, 3\pm 2i\}$
(4) $x\in\{3, \frac{-1\pm i\sqrt{3}}{2}\}$

32.2 CONJUGATE ROOT THEOREM

QUESTIONS 32.2.

Q2. (1) x^2+4
(2) $x^2-2x+5i$
(3) x^4+13x^2+36
(4) $x^4-2x^3+2x^2-10x+25$

Q3. $2\pm i, \pm 3i$

Q4. $-1\pm 2i, -1\pm 2i\sqrt{6}$

Q5. $-3\pm i\sqrt{3}, 1\pm i\sqrt{3}$

Q6. $2\pm i\sqrt{2}, \frac{1}{2}\pm i\frac{\sqrt{3}}{2}$

Q7. $-3\pm 2i, -\frac{1}{2}\pm i$

Q8. $\pm i, 2\pm i, -2\pm i$

Q9. $\pm i, 1\pm i$

Q10. $2\pm i, 1\pm 2i$

Q11. $\pm 3i, 1\pm i\sqrt{2}$

Q12. (i) —
(ii) —
(iii) $z^4+4z^2+6+4z^{-2}+z^{-4}$
(iv) $\cos^4\theta=\frac{1}{8}\cos 4\theta+\frac{1}{2}\cos 2\theta+\frac{3}{8}$
(v) $\frac{1}{4}+\frac{3\pi}{32}$

Q13. (i) —
(ii) —
(iii) $z^5-5z^3+10z-\frac{10}{z}+\frac{5}{z^3}-\frac{1}{z^5}$
(iv) $\frac{1}{16}\sin 5\theta-\frac{5}{16}\sin 3\theta+\frac{5}{8}\sin\theta$
(v) $\frac{8}{15}$

Q14. —

PART IX

STATISTICS

CHAPTER 33

POPULATION, SAMPLE, TYPES OF DATA

CHAPTER 34

Presentation of data

Questions 34.1.

Q1. mean height $\approx 160cm$

h	f	$h \times f$
152	4	608
153	2	306
157	3	471
160	3	480
163	6	978
165	3	495
168	3	504

Q2. (i) mode is $163cm$

(ii) $Q_0 = 152cm$

(iii) $Q_4 = 168cm$

(iv) range is $16cm$

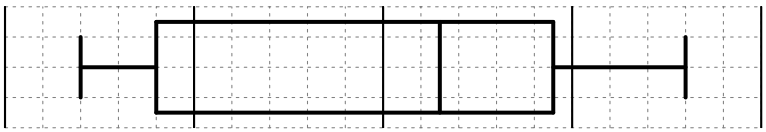
(v) median is $Q_2 = 161.5cm$

(vi) $Q_1 = 154cm$

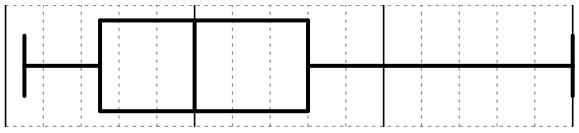
(vii) $Q_3 = 164.5cm$

(viii) I.Q.R. $= 10.5cm$

Q_0	Q_1	Q_2	Q_3	Q_4
152	154	161.5	164.5	168



Q3. (i) 70 (ii) 65 (iii) 76 (iv)



Q4. (i)

no of sand.	freq.	cum.freq.
0	6	6
1	14	20
2	48	68
3	17	85
4	9	94
5	6	100

(ii) median $= 2$, $Q_1 = 2, Q_3 = 3$

(iii)

(iv) TRUE, FALSE, FALSE, FALSE, TRUE

Q5. FALSE, FALSE, FALSE, FALSE

CHAPTER 35

Measures of tendency, measures of spread

Questions 35.1.

Q1. (i) mean = 172.5 (ii) range = 22

Q2. 164, 178

Q3. **SET A**: mean= 24.5, st.dev.= 3.64, **SET B**: mean= 24.5, st.dev.= 1.71

Q4. (i) — (ii) A: 46.2, B: 15.8

Q5.

$\mu-2\sigma$	$\mu-\sigma$	μ	$\mu+\sigma$	$\mu+2\sigma$
397	425	453	481	509

(i) 68% (ii) 95% (iii) 2.5% (iv) 16% (v) 97.5% (vi) 97.5%

Q6.

$\mu-2\sigma$	$\mu-\sigma$	μ	$\mu+\sigma$	$\mu+2\sigma$
20	24	28	32	36

(i) 68% (ii) 95% (iii) 16% (iv) 2.5% (v) 84% (vi) 97.5%

CHAPTER 36

CUMULATIVE FREQUENCY

QUESTIONS 36.1.

Q1.

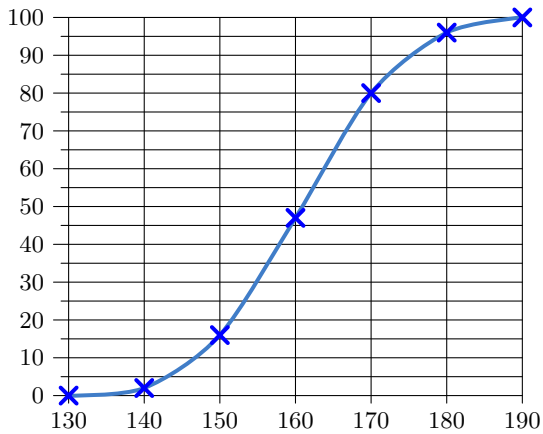
h	f	x	xf
$130 < h \leq 140$	2	135	270
$140 < h \leq 150$	14	145	2030
$150 < h \leq 160$	31	155	4805
$160 < h \leq 170$	33	165	5445
$170 < h \leq 180$	16	175	2800
$180 < h \leq 190$	4	185	740

mean height $\approx 160.9cm$

Q2.

height	freq.	cum.freq.
$h \leq 140$	2	2
$h \leq 150$	14	16
$h \leq 160$	31	47
$h \leq 170$	33	80
$h \leq 180$	16	96
$h \leq 190$	4	100

Q3.



(i) median ≈ 162 (ii) $Q_1 \approx 154$ (iii) $Q_3 \approx 168$ (iv) $I.Q.R. \approx 14$

Q4. (i) median: $18 - 19cm$ (iii) $I.Q.R. = 8$

(ii) $Q_1 = 15, Q_3 = 23$ (iv) $a = 12, b = 28$

PART X

PROBABILITY

CHAPTER 37

COUNTING PRINCIPLES

QUESTIONS 37.1.

Q1. (1) 45
(2) 650
(3) 1640
(4) 560
(5) 1320
(6) 650
(7) 84
(8) 1001

Q2. 4

Q3. (1) 4
(2) 3
(3) 6
(4) 6

Q4. —

Q5. —

Q6. (i) Use $(1+1)^n$
(ii) 2^n

Q7. (1) 120
(2) 3125

Q8. (1) 729
(2) 90
(3) $3^6-(3\times2^6-3)=540$

Q9. (1) 60
(2) 720

Q10. (1) 16807
(2) 2520
(3) 9604
(4) 1440

Q11. (1) 14406
(2) 2160
(3) 8232
(4) 1260

Q12. 240

Q13. 480

Q14. 288

Q15. (1) 288
(2) 576
(3) 144

Q16. (1) 1287
(2) 1266
(3) 1260
(4) 945

Q17. (1) 5040
(2) 1440
(3) 3600

Q18. (1) 120
(2) 12
(3) 36

Q19. 720

Q20. 210

Q21. 34650

Q22. (1) 495
(2) 210
(3) 285

Q23. (1) 362880
(2) 282240
(3) 1728

Q24. (1) 15120
(2) 4200
(3) 720
(4) 480
(5) 120
(6) 3024
(7) 504
(8) 360
(9) 144
(10) 72

Q25. (1) 2598960

(2)
- **Straight flush** - 36
- **Four of a Kind** - 624
- **Full House** - 3744
- **Flush** - 5112
- **Straight** - 10200
- **3 of a Kind** - 54912
- **Two Pair** - 123552
- **One Pair** - 1098240
- **High Cards** - 1302540

Q26. (1) 1677106640
(2) 1086008
(3) 40
(4) $\approx 4.42\times10^{11}$
(5) $\approx 2.32\times10^{11}$
(6) $\approx 1.07\times10^{11}$

Q27. (1) $4^4 = 256$ (2) $5^5 = 3125$ (3) $5^4 = 625$ (4) $4^5 = 1024$ (5) 24 (6) 24 (7) 120 (8) 0 (9) 0 (10) 240

CHAPTER 38

Introduction to probability

38.1 Concept of probability

38.2 Formal approach

38.3 Axioms of probability

38.4 Exemplary simple probability problems

38.5 Counting principles

Questions 38.1.

Q1. (i) $\frac{1}{2}$ (ii) $\frac{1}{3}$ (iii) $\frac{1}{3}$

Q2. 0.3

Q3. (i) $\frac{1}{7}$ (ii) $\frac{4}{7}$ (iii) $\frac{5}{7}$

Q4. (i) 0.678 (ii) 0.283 (iii) 0.0391 (iv) 0.407 (v) 0.497 (vi) 0.0957 (vii) 0.22 (viii) 0.26 (ix) 0.52

CHAPTER 39

Conditional probability

39.1 Combined events

Questions 39.1.

Q1. all 4 are true

Q2. (i) 0.61 (ii) 0.11 (iii) 0.22 (iv) 0.39

Q3. (i) 0.9 (ii) 0.2 (iii) 0.2 (iv) 0.9

Q4. (i) 0.85 (ii) 0.25 (iii) 0.6 (iv) 0.2 (v) 0.8 (vi) 0.6

Q5. (i) 0.35 (ii) 0.15 (iii) 0.85

Q6. (i) 0.35 (ii) 0.9 (iii) 0.35

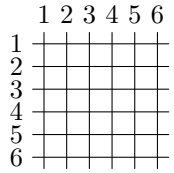
Q7. (i) p^2 (ii) $p - p^2$ (iii) $1 - p^2$

Q8. (i) $\frac{1}{7}$ (ii) $\frac{4}{7}$ (iii) $\frac{3}{7}$

Q9. (i) $\frac{2}{13}$ (ii) $\frac{4}{13}$

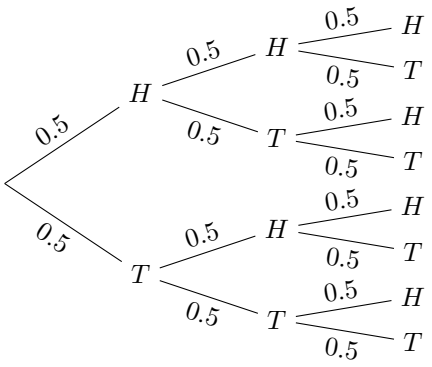
Q10. (i) $\frac{9}{25}$ (ii) $\frac{13}{25}$

Q11. (i)



(ii) $\frac{1}{36}$
(iii) $\frac{1}{6}$
(iv) $\frac{5}{12}$
(v) $\frac{1}{6}$
(vi) $\frac{7}{12}$

Q12. (i)



$$U = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

(ii) $\frac{3}{8}$
(iii) $\frac{3}{8}$
(iv) $\frac{1}{2}$
(v) $\frac{1}{4}$

39.2 Conditional probability

39.3 Independent events

39.4 The law of total probability

39.5 Bayes' theorem

Questions 39.2.

Q13. (i) 0.2 (ii) $\frac{2}{3}$ (iii) $\frac{2}{7}$ (iv) $\frac{5}{7}$ (v) NO

Q14. (i) 0.8 (ii) 0.6 (iii) 0.5 (iv) 0.5 (v) YES

Q15. (i) 0.3 (ii) 0.4 (iii) 0.75 (iv) 0.6 (v) YES

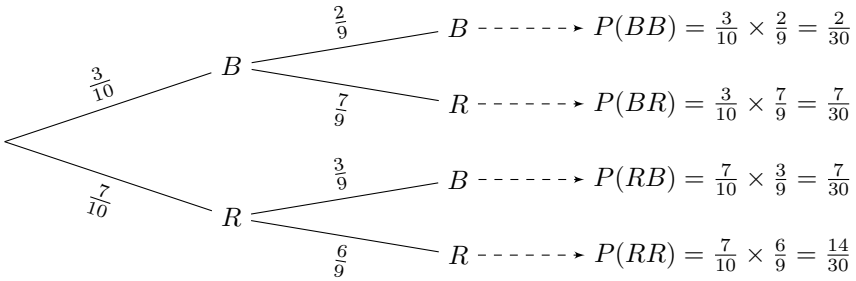
Q16. (i) 0.7 (ii) $\frac{6}{7}$ (iii) $\frac{6}{13}$ (iv) $\frac{7}{13}$ (v) NO

Q17. (i) 0 (ii) 0 (iii) 0 (iv) 0.5

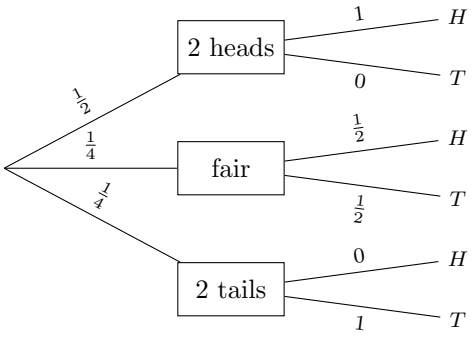
Q18. (i) 0.89 (ii) 0.2 (iii) 0.45 (iv) 0.55

Q19. (i) 0.24 (ii) 0.5 (iii) 0.52 (iv) 0.5

Q20. (i)



(ii) $\frac{7}{15}$
(iii) $\frac{7}{10}$
(iv) $\frac{7}{30}$
(v) $\frac{14}{15}$
(vi) $\frac{7}{15}$
(vii) $\frac{7}{9}$
(viii) $\frac{6}{9}$
(ix) $\frac{7}{10}$
(x) $\frac{1}{2}$



(ii) $\frac{5}{8}$
(iii) $\frac{1}{5}$

Q21. (i)

Q22. (i) $\frac{4}{9}$ (ii) $\frac{4}{9}$

Q23. (i) $\frac{1}{3}$ (ii) $\frac{4}{9}$

Q24. (i) $\frac{31}{45}$ (ii) $\frac{1}{5}$ (iii) $\frac{5}{9}$ (iv) $\frac{2}{9}$

Q25. (i) $\frac{1}{2}$ (ii) $\frac{1}{2}$ (iii) $\frac{1}{4}$ (iv) 1 (v) $\frac{1}{3}$

Q26. (i) $\frac{5}{36}$ (ii) $\frac{1}{5}$ (iii) $\frac{2}{5}$ (iv) $\frac{4}{5}$ (v) $\frac{2}{15}$

Q27. (i) 0.0382 (ii) 0.962 (iii) 2.11×10^{-5}

Q28. —

Q29. (i) $\frac{1}{20}$ (ii) $\frac{7}{20}$ (iii) $\frac{4}{7}$

Q30. (i) 0.48 (ii) 0.1 (iii) 0.64

Q31. (i) $\frac{1}{8}$ (ii) $\frac{3}{8}$ (iii) $\frac{7}{8}$ (iv) $\frac{3}{7}$

Q32. (i) $\frac{8}{27}$ (ii) $\frac{4}{9}$ (iii) $\frac{26}{27}$ (iv) $\frac{6}{13}$

Q33. (i) 0.45 (ii) 0.25 (iii) $\frac{1}{3}$ (iv) $\frac{2}{11}$

Q34. (i) 0.041 (ii) $\frac{6}{41}$ (iii) $\frac{35}{41}$

Q35. (i) $\frac{6}{35}$ (ii) $\frac{23}{35}$ (iii) $\frac{18}{35}$ (iv) $\frac{14}{23}$ (v) $\frac{6}{23}$

Q36. (i) $\frac{9}{35} \approx 0.257$,
(ii) $\frac{19}{210} \approx 0.0905$,
(iii) $\frac{13}{35} \approx 0.371$,
(iv) $\frac{5}{26} \approx 0.192$,
(v) $\frac{35}{66} \approx 0.53$.

Q37. (i) 0.09, (ii) $\frac{2}{9} \approx 0.222$, (iii) $\frac{17}{91} \approx 0.187$.

Questions 39.3.

Q1. It is twice better to switch.

Q2. $\frac{1}{5}$

Q3. $\frac{3}{8}$

Q4. 0.179

Q5. (i) $\frac{1}{18}$
(ii) $\frac{1}{36}$

Q6. 5

Q7. $\frac{13}{20}$

Q8. (i) $\frac{16}{55}$
(ii) $\frac{4}{11}$
(iii) $\frac{19}{55}$

CHAPTER 40

Discrete random variables

40.1 Discrete random variables

Questions 40.1.

Q1. 0.45

Q2. 0.1

Q3. (i) 0.5 (iii) 0.75 (v) 0.75
(ii) 0 (iv) 1

Q4. (i) $\{0, 1, 2\}$ (ii)

d	0	1	2
$P(D = d)$	0.1	0.6	0.3

(iii) $\frac{1}{3}$

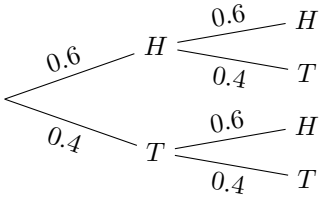
Q5. (i) $\frac{7}{12}$ (ii) $\frac{1}{2}$ (iii) $\frac{3}{7}$

Q6. (i)

x	2	3	4	5	6
$P(X = x)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{2}{9}$	$\frac{1}{9}$

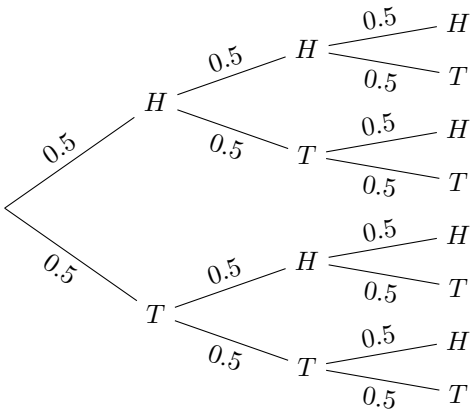
(ii) $\frac{5}{8}$
(iii) $\frac{2}{3}$
(iv) $\frac{1}{3}$ both

Q7. (i)



(ii) 0.64
(iii) $\frac{3}{7}$
(iv) (a) 0.4
(b) 0.24
(c) 0.144
(v) 0.0187
(vi) 0.36
(vii) 0.4704
(viii) 0.64
(ix) 10

Q8. (i)



(ii)

t	0	1	2	3
$P(T = t)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(iii) $\frac{1}{2}$
(iv) $\frac{3}{4}$

Q9. (i)

y	0	2
$P(Y = y)$	$\frac{7}{12}$	$\frac{5}{12}$

(ii) $\frac{7}{12}$

Q10. (i)

b	0	1	2
$P(B = b)$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$

(ii) $\frac{2}{3}$
(iii) $\frac{1}{2}$

Q11.

x	0	1	2
$P(X = x)$	$\frac{2}{5}$	$\frac{8}{15}$	$\frac{1}{15}$

Q12. (i)

x	0	1	2	3	4	5
$P(X = x)$	0.0298	0.192	0.383	0.298	0.0894	0.00813

(ii) 0.970
(iii) 0.492

Q13. (i)

l	0	1	2	3
$P(L = l)$	0.399	0.461	0.132	0.00877

(ii) 0.601
(iii) 0.219

Q14. (i) 10 (ii) (a) $\frac{7}{10}$ (b) $\frac{7}{9}$

Q15. (i) $\frac{1}{26}$ (ii) (a) $\frac{25}{26}$ (b) $\frac{10}{11}$

Q16. (i)

x	0	0.5	1	1.5	2	2.5	3	5
$F(x)$	0	0	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{4}{9}$	$\frac{4}{9}$	1	1

(ii)

x	0	0.5	1	1.5	2	2.5	3	5
$P(X = x)$	0	0	$\frac{1}{9}$	0	$\frac{1}{3}$	0	$\frac{5}{9}$	0

40.2 Expected value of a discrete random variable
40.3 The meaning of expected value
40.4 Properties of expected value of a discrete random variable
40.5 Variance of a discrete random variable
40.6 Properties of variance of a discrete random variable

Questions 40.2.

Q17. (i) 2.85
(ii) 1.43

Q18. (i) 4.65
(ii) 19.4
(iii) -6.3
(iv) 8.82
(v) 19.85
(vi) 1740

Q19. (i) 0.5
(ii) 1.75
(iii) 2
(iv) 7
(v) 2
(vi) 2.5
(vii) 1.75
(viii) $-\frac{1}{3}$
(ix) $\frac{25}{36}$

Q20. $E(B) = 1, Var(B) = \frac{1}{3}$

Q21. $E(X) = \frac{2}{3}, Var(X) = \frac{16}{45}$

Q22. $E(X) = 2.25, Var(X) \approx 0.977$

Q23. (i) 10
(ii) $E(X) = 3, Var(X) = 1$

Q24. (i) $\frac{1}{26}$
(ii) $E(X) \approx 11.9, Var(X) \approx 24.8$

Q25. $E(X) = 7, Var(X) = \frac{35}{6}$

Q26. 12.25

Q27. $E(X) = 1.5, Var(X) \approx 0.866$

Q28. $E(X) = 0.6, Var(X) = \frac{28}{75}$

Q29. $E(X) = 1, Var(X) = 0.75$

Q30. $E(X) = \frac{2}{3}, Var(X) = \frac{4}{9}$

Q31. $E(X) = \frac{8}{9}, Var(X) = \frac{44}{81}$

Q32. No, it's a loosing game.

Q33. should win $\$\frac{8}{3}$,
i.e. the game cannot be fair

Q34. \$1.50

Q35. (i) 1
(ii) 4
(iii) 1.25

Q36. (i) 2.68
(ii) 8.91
(iii) 1.68

chapter 41

Binomial distribution

Questions 41.1.

Q1. (i) 0.118
(ii) 7.29×10^{-4}
(iii) 0.324
(iv) 0.580
(v) 0.579
(vi) 1.8
(vii) 1.12

Q2. (i) 5.08×10^{-10}
(ii) 4.24×10^{-32}
(iii) 5.49×10^{-4}
(iv) 0.331
(v) 0.838
(vi) 18
(vii) 3.55

Q3. (i) 0.0000276
(ii) 0.154
(iii) 0.249

Q4. (i) 9.09×10^{-13}
(ii) 0.00992
(iii) 0.0409
(iv) 0.00317
(v) 5
(vi) 15

Q5. (i) 0.282
(ii) 10^{-12}
(iii) 0.111
(iv) 20

Q6. (i) 0.329
(ii) 0.649
(iii) 0.351
(iv) 0.320
(v) 0.338

(vi) 0.493
(vii) 0.610
(viii) 0.307

Q7. (i) $\frac{1}{3}$
(ii) 0.273
(iii) 0.961
(iv) 0.0658
(v) at least 6

Q8. (i) $p = 0.3$, $n = 20$
(ii) 0
(iii) 0.107
(iv) 0.238
(v) 0.113
(vi) 0.392
(vii) 0.346
(viii) 0.209

Q9. (i) 0.100
(ii) 0.0865
(iii) 0.973
(iv) 0.433
(v) 0.00317
(vi) 0.320
(vii) 0.505

CHAPTER 42

POISSON DISTRIBUTION

QUESTIONS 42.1.

Q1. (i) 0.223
(ii) 0.603
(iii) 0.380
(iv) 0.959
(v) 0.829
(vi) 0.381
(vii) 0.614
(viii) 0.359
(ix) 0.830
(x) 3
(xi) 3
(xii) 3.2

Q2. (i) 8.78
(ii) 0.00592
(iii) 0.130
(iv) 0.993
(v) 0.128
(vi) 0.993
(vii) 9
(viii) 8
(ix) 2.96

Q3. (i) 4.20
(ii) 0.589
(iii) 0.247
(iv) 0.395
(v) 0.789
(vi) 0.679

Q4. (i) 2.80
(ii) 0.222
(iii) 0.308
(iv) 0.935
(v) 0.300
(vi) 0.0264

Q5. (i) 9.12×10^{-4}
(ii) 2.75×10^{-5}
(iii) 0.497
(iv) 0.966
(v) 0.156
(vi) 0.691
(vii) 0.0114
(viii) 0.514
(ix) 0.126
(x) 0.486

Q6. (i) 0.670
(ii) 0.551
(iii) 0.813

Q7. (i) 7.35×10^{-11}
(ii) 1
(iii) 0.813

Q8. (i) 0.247
(ii) 0.308
(iii) 0.779
(iv) 0.458

Q9. (i) 0.435
(ii) 0.203
(iii) 0.137
(iv) 0.134

Q10. (i) 0.393
(ii) 0.0144
(iii) 4.54×10^{-5}
(iv) 5.89×10^{-4}
(v) 0.226
(vi) 0.00426

Q11. (i) 0.741
(ii) 2.04×10^{-5}
(iii) 0.655
(iv) 0.474

Q12. (i) 0.0380
(ii) 0.00149
(iii) 0.956
(iv) 0.998
(v) 0.958
(vi) 0.958
(vii) 0.159
(viii) 0.683
(ix) 0.325

Q13. (i) 0.997
(ii) 0.351
(iii) 0.0596
(iv) 0.999
(v) 2.06×10^{-9}
(vi) 4.12×10^{-8}
(vii) 1(0.99992...)

Q14. (i) 0.4
(ii) 0.191
(iii) 7

CHAPTER 43

CONTINUOUS RANDOM VARIABLES

QUESTIONS 43.1.

Q1. (i) —
(ii) $3a$
(iii) $\frac{1}{3}$
(iv) see below
(v) $\frac{2}{3}$
(vi) $\frac{2}{3}$
(vii) no mode
(viii) 1
(ix) 1
(x) $\frac{3}{4}$
(xi) $\frac{1}{6}$ and $\frac{1}{2}$
(xii) $\begin{cases} 0 & \text{for } x < -0.5 \\ \frac{1}{3}(x + \frac{1}{2}) & \text{for } -0.5 \leq x \leq 2.5 \\ 1 & \text{for } x > 2.5 \end{cases}$

Q2. (i) —
(ii) $\frac{4}{b}$
(iii) 4
(iv) see below
(v) 0
(vi) 1
(vii) $4-\sqrt{5}$
(viii) $\frac{11}{6}$
(ix) $\frac{11}{36}$
(x) $\begin{cases} 0 & \text{for } x<1 \\ -\frac{1}{8}(x^2-8x+7) & \text{for } 1\le x\le 3 \\ 1 & \text{for } x>3 \end{cases}$

Q3. (i) —
(ii) $\ln(c+1)$
(iii) $e-1$
(iv) see below
(v) $\ln 2$
(vi) 0
(vii) $\sqrt{e}-1$
(viii) $e-2$
(ix) $\frac{-e^2+4e-3}{2} \approx 0.242$
(x) $\begin{cases} 0 & \text{for } x<0 \\ \ln(x+1) & \text{for } 0\le x\le e-1 \\ 1 & \text{for } x>e-1 \end{cases}$

Q4. (i) —
(ii) $\frac{10k}{3}$
(iii) 0.3
(iv) see below
(v) 0.65
(vi) 1.5
(vii) c.1.76
(viii) 1.8
(ix) 0.24

Q5. (i) —
(ii) $2s$
(iii) $\frac{1}{2}$
(iv) see below
(v) 1
(vi) $\frac{1}{2}$
(vii) $\ln 3$
(viii) $\ln 2$
(ix) $\frac{3}{2}\ln 3 - 1$
(x) 0.0948

Q6. (i) —
(ii) $2t$
(iii) $\frac{1}{2}$
(iv) see below
(v) 0.518
(vi) 0
(vii) 0
(viii) 0
(ix) 0.352

Q7. (i) —
(ii) $\frac{r}{3}$
(iii) 3
(iv) see below
(v) $\frac{1}{2}$
(vi) $\frac{1}{2}$
(vii) $\frac{1}{2}$
(viii) $\frac{1}{2}$
(ix) 0.0125

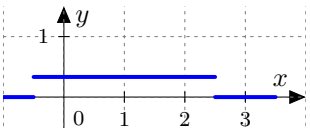
Q8. (i) —
(ii) 1
(iii) —
(iv) see below
(v) 0
(vi) $\ln 2$
(vii) 1
(viii) 1

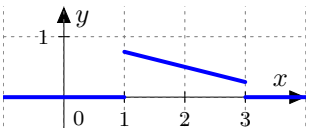
Q9. (i) $\frac{1}{9}$ (ii) 1.77 (iii) $\frac{3}{\sqrt{2}} \approx 2.12$ (iv) 1.82 (v) 0.691

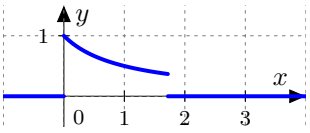
Q10. (i) 4 (ii) 1.21 (iii) 1.18 (iv) 0.917 (v) 1

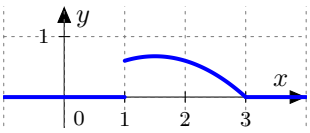
Q11. (i) $a = 0.7, b = 0.2$ (ii) $\frac{3}{14} \approx 0.214$ (iii) $\frac{\sqrt{14}+1}{7} \approx 0.677$

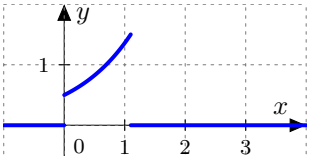
GRAPHS:

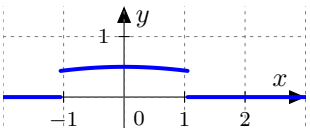
Q1.


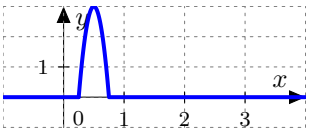
Q2.


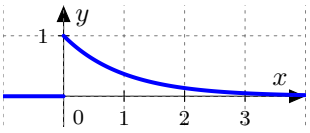
Q3.


Q4.


Q5.


Q6.


Q7.


Q8.


CHAPTER 44

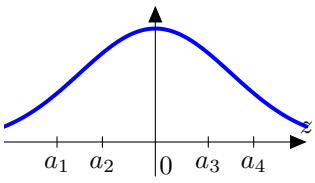
NORMAL DISTRIBUTION

QUESTIONS 44.1.

Q1. (i) (1) 0.692
(2) 0.998
(3) 1
(4) 0.5
(5) 0.268
(6) 0.032
(7) 0.841
(8) 0.979
(9) 0.813

(10) 0.0108
(11) 0.0793
(12) 0.00135
(13) 0.153
(14) 0.138
(15) 0.206
(16) 0.683
(17) 0.695
(18) 0.856

(ii) (1) 0.345
(2) 0.931
(3) 3.09
(4) -0.192
(5) -0.504
(6) -1.41
(7) -0.643
(8) -0.482
(9) -1.28
(10) 0.675
(11) 0.410
(12) 1.96



Q2. (i)

(ii) (1) $1 - p_1$
(2) $1 - p_4$
(3) $p_4 - p_2$
(4) $p_3 - p_2$
(5) p_1
(6) $1 - p_4$
(7) $2p_3 - 1$
(8) $1 - 2p_1$
(9) $p_4 - p_3$
(10) $p_2 - p_1$
(11) $p_3 + p_4 - 1$
(12) $p_3 + p_4 - 1$

(iii) (1) $-\infty, a_1$
(2) $-\infty, a_4$
(3) a_1, a_2
(4) $a_1, +\infty$
(5) $a_3, +\infty$
(6) a_1, a_4
(7) a_2, a_3
(8) a_3, a_4

Q3. (1) 0
(2) 1
(3) $P(Z < 1) \approx 0.841$
(4) $P(Z < \frac{a-5}{2}), a \approx 4.65$

Q4. (1) 0.841
(2) 0.933
(3) 0.309
(4) 0.629
(5) 0.212
(6) 0.341
(7) 0.323
(8) 0.657
(9) 0.497

Q5. (1) 0.988
(2) 0.749
(3) 0.027
(4) 0.922
(5) 0.254
(6) 0.377
(7) 0.251
(8) 0.716
(9) 0.652

Q6. (1) 0.742
(2) 0.298
(3) 0.742
(4) 0.115
(5) 0.324
(6) 0.505

Q7. (1) 8.36
(2) 9.19
(3) 9.31
(4) 5.81
(5) 4.94
(6) 2.6

Q8. (1) 65.8
(2) 65.6
(3) 64.6
(4) 61.4
(5) 62.0
(6) 60.4

Q9. (1) 40.1%
(2) 1.22%
(3) 93.1%
(4) $m = 1005g$
(5) $d = 999g$

Q10. (1) 59.3%
(2) 0.0228
(3) 0.53
(4) 4 min.15 sec.
(5) 3 min.29 sec.

Q11. (1) $h_1 \approx 26.8cm$
(2) $h_2 \approx 29.8cm$
(3) 3.08

Q12. (1) 0.276
(2) 6.61×10^{-6}
(3) 0.516
(4) 503

Q13. (1) 43.7
(2) 1.22%
(3) 62.3 and higher
(4) 4.78×10^{-6}

Q14. $172cm$

Q15. $57.3kg$

Q16. 1.17

Q17. 1.69

Q18. $\mu = 60.5, \sigma = 17.4$

Q19. $\mu = 70.6, \sigma = 27.4$

Q20. $\mu = 1.67, \sigma = 0.658$

Q21. $\mu = 858, \sigma = 111$

Q22. $\mu = 17.7, \sigma = 3.37$

Q23. $\mu = 20, \sigma = 10.4$

Q24. $\mu = 16.3, \sigma = 5.69$

Q25. $\mu = 131, \sigma = 11.3$

Q26. $\mu = 25, \sigma = 7.72$